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Abstract

In the present paper, we examine the conditions for synchronization of a delayed stochastic model of earthquake nucleation, where each block is under the impact of its previous state and the neighboring blocks and under the stochastic perturbation. As a result of our work, we obtain a sufficient condition for the exponential mean square stability of the synchronization. Results obtained indicate that the bi-directionally symmetric coupling induces the synchronization much less efficiently than the uni-directionally symmetric one. From the seismological viewpoint, it seems that different parts (patches, segments) of the same fault or neighboring faults could exhibit synchronized activity under certain conditions. This could further induce the occurrence of much stronger earthquakes or could extend the duration of earthquakes, which could further have more severe consequences at the ground surface (including the effects on structures and people) compared to the case of effects of unsynchronized motion of different segments of the same fault or group of neighboring faults.

Keywords (separated by '-'): earthquake nucleation - fault motion - synchronization

Synchronized Dynamics of Earthquake Fault Motion

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Abstract. In the present paper, we examine the conditions for synchronization of a delayed stochastic model of earthquake nucleation, where each block is under the impact of its previous state and the neighboring blocks and under the stochastic perturbation. As a result of our work, we obtain a sufficient condition for the exponential mean square stability of the synchronization. Results obtained indicate that the bi-directionally symmetric coupling induces the synchronization much less efficiently than the uni-directionally symmetric one. From the seismological viewpoint, it seems that different parts (patches, segments) of the same fault or neighboring faults could exhibit synchronized activity under certain conditions. This could further induce the occurrence of much stronger earthquakes or could extend the duration of earthquakes, which could further have more severe consequences at the ground surface (including the effects on structures and people) compared to the case of effects of unsynchronized motion of different segments of the same fault or group of neighboring faults.

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1 Introduction

Earthquakes represent natural phenomena that pose the greatest threat to society. There are many examples from the recent past where earthquakes caused significant material damage and led to massive loss of human lives. For instance, the most recent earthquake M 6.8 that hit Morocco on 8th September 2023 affected more than 320.000 people, according to the World Health Organization [1]. Also, M 7.7 earthquake occurred in Turkey on 6th February 2023 affecting 16.3 million people according to the International Federation of Red Cross and Red Crescent Societies [2]. Considering the size of the immediate and potential threat of earthquakes, it is of great practical significance to examine the conditions of earthquake nucleation, i.e. especially conditions that lead to the

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occurrence of strong earthquakes. In our previous research, we thoroughly investigated the conditions for the occurrence of unstable fault dynamics that lead to earthquake nucleation. Our previous research included the analysis of the effect displacement delay [3], external pulse [4], random noise [5], and colored noise [6] on dynamics of spring-block models, composed of a single block, two blocks, and array of blocks with rate- and state-dependent friction and only rate dependent friction. Nevertheless, none of our previous research dealt with the conditions for possible synchronization of motion between the neighboring parts of the same fault, or the nearby faults. It seems that the identification of such conditions could have significant implications for the size of the area affected by a generated earthquake. In particular, if two or more neighboring faults synchronize – a much larger area will be affected by seismic motion, compared to the conditions when instability is generated only by a movement along a single fault.

The concept of potential synchronization in the context of general earthquake motion has long been a subject of inquiry within seismology. Among the early pioneers in this field, Scholz [7] stands out for his exploration of the conditions conducive to large earthquake triggering, clustering, and synchronization of faults. His seminal work highlighted the significance of small earthquake stress drops and triggering delay times relative to the natural recurrence time of earthquakes, suggesting that faults must be near the ends of their seismic cycles for synchronization to occur. Paleoseismological data further supported this notion by revealing clusters of ruptures across several faults within the same regions, separated by extended quiescent periods, indicative of synchronization between faults. Building upon these foundational insights, subsequent researchers, such as Meng and Pengcheng [8], delved into the conditions for synchronization of earthquake cycles within adjacent segments of oceanic transform faults. Through numerical modeling within the framework of rate-and-state friction, they investigated the dynamics of earthquake cycles on the west Gofar fault, East Pacific Rise. Their findings demonstrated that synchronization could be achieved through mechanisms such as after slip or post-seismic creep in barrier patches, or even through static stress transfer. Further contributions to this field include analyses of different earthquake models to discern the conditions for synchronization. For instance, Chelidze et al. [9] explored the synchronization potential of the spring-slider system operating in a stick-slip regime, akin to active tectonic faults generating earthquakes. Their investigations revealed instances of high-order synchronization of stick-slip events under the influence of weak electromagnetic or mechanical periodic forcing. Similarly, Cherepanstev [10] examined the possibility of synchronization within the dissipative cellular model of earthquakes, uncovering distinct modes of synchronous drop formation simulating earthquake events. Collectively, these studies have advanced our understanding of the complex dynamics underlying earthquake synchronization, shedding light on the diverse mechanisms and conditions that contribute to this phenomenon. By elucidating the intricacies of synchronization processes, researchers aim to enhance our ability to predict and mitigate the impacts of seismic events, ultimately bolstering earthquake resilience and hazard management strategies.

In the present paper, we consider a microscopic earthquake model, with delayed coupling and background noise. Such a model is considered discrete with the main ingredients being the individual blocks (different neighboring faults or patches of the same fault). For such a model, we provide a sufficient condition for synchronization.

The paper is structured as follows. In Sect. 2 we provide a brief description of the examined model. In Sect. 3 we provide the main results of the conducted analysis. Conclusions are provided in the final section, together with the directions for further research.

2 Starting Model

The modeling of earthquake fault motion through the analysis of spring-block models traces its roots back to the pioneering work of Burridge and Knopoff [11] in the latter half of the 20th century. Their seminal contribution involved proposing the first mechanical model to describe earthquake fault motion, alongside formulating the corresponding governing equations. Since then, this chain model of earthquake nucleation has served as a fundamental framework for studying seismogenic fault motion. Building upon this foundational work, subsequent investigations have delved deeper into the dynamics of such models, primarily focusing on examining the stability of solutions. Through a series of studies conducted by various researchers, including those referenced in literature [12–14], efforts have been made to elucidate the intricate dynamics governing earthquake fault motion within this framework. In our prior research endeavors, we have further expanded the horizons of this field by exploring the impact of two critical factors: time delay and random noise [5]. Specifically, we investigated the effects of time delay in the positions of the blocks comprising the array, drawing inspiration from the Stribeck effect [3] and considering the delayed interaction among different parts of the same fault or neighboring faults [15]. This consideration of time delay reflects the nuanced nature of fault dynamics, where temporal delays can play a significant role in shaping the evolution of seismic events. Moreover, we delved into the effects of both random noise and colored noise [6] within the context of a rate-dependent friction law. These explorations aimed to unravel the influence of stochastic processes on earthquake fault motion, acknowledging the inherent variability and unpredictability characteristic of seismic phenomena. By integrating these factors into our analysis, we aim to contribute to a more comprehensive understanding of earthquake fault dynamics, shedding light on the complex interplay between mechanical processes, temporal delays, and stochastic influences. Ultimately, our endeavors seek to advance the field's ability to accurately model and predict seismic events, thereby enhancing earthquake preparedness and hazard mitigation efforts. One should note that delayed interaction among different units of the natural system has also been investigated for the sake of the analysis of landslide dynamics [16].

In the present paper, we examine the general model of earthquake fault motion, expressed by scalar delay-differential equations (DDE) of the following form:

$$\begin{aligned} du_i(t) &= \left(-au_i(t) + bf\left(u_i(t)\right) + cg\left(u_i^{\tau_1}(t)\right)\right)dt - \left(d_1\left(u_i^{\tau_2}(t) - u_{i-1}^{\tau_2}(t)\right) + \right. \\ & \left. d_2\left(u_i^{\tau_2}(t) - u_{i+1}^{\tau_2}(t)\right)\right)dt + u_i(t)\sqrt{2D}dW \end{aligned} \tag{1}$$

where $i = 2,3\ldots, N\text{-}1$, $u^{\tau_1}(t) \equiv u(t - \tau_1)$, f and g are quite general nonlinear functions and a, b, c are parameters. Equation (1) is considered as the starting equation for which we provided conditions of synchronization.

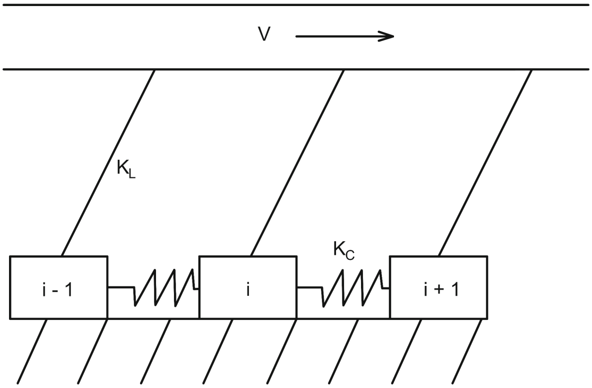

Fig. 1. Illustration of the examined phenomenological spring-block model of earthquake fault motion.

AQ2 Regarding the noise effect, we shall always assume that dW, formally written as $dW = \xi(t)dt$ is the stochastic increment of the Wiener process $\xi(t)$ for which: $E(\xi) = 0$*concerning* he stochastic process. The increments satisfy $E(dW) = 0, dWdW = dt$ (Fig. 2).

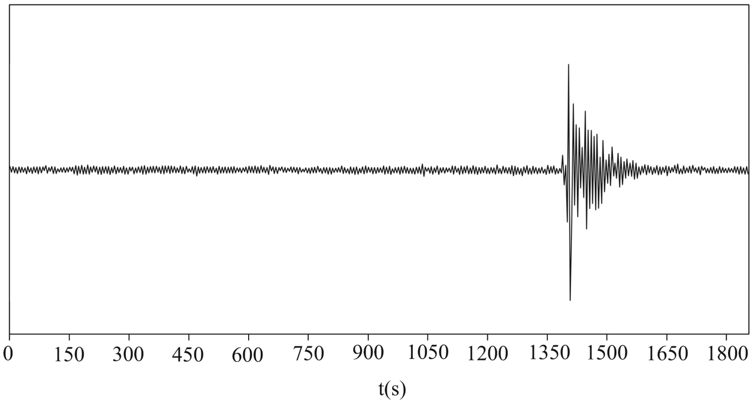

Fig. 2. Example of recorded background ambient noise at an accelerograph station: Zavoj in Serbia (seismo.gov.rs)

Over the years, the field of geological engineering has often sidelined the consideration of noise, perceiving it as inconsequential due to its seemingly minuscule impact compared to the monitored processes and its higher frequency of events of interest. Nevertheless, recent strides in seismic recording technologies have unveiled a nuanced understanding of noise's significance, particularly in systems approaching critical stability thresholds like earthquakes and landslides. This newfound awareness underscores the intricate interplay between noise and various environmental factors, such as precipitation patterns, fluctuations in groundwater levels, and the relentless shifts in tectonic activity, all of which can exacerbate the effects of noise on geological phenomena.

Defined as subtle yet persistent seismic vibrations within the microseism band, noise emanates from a labyrinth of intricate mechanisms whose origins often elude straightforward explanation. Nonetheless, through the deployment of cutting-edge recording instruments and sophisticated analytical methodologies, scientists have been able to peel back the layers of complexity surrounding background noise. As a result, a clearer picture has emerged, revealing its peculiar traits of quasi-randomness, enduring constancy, and dynamic amplitude fluctuations contingent upon factors like frequency, spatial positioning, and temporal variations.

Within this intricate tapestry of geological acoustics, Webb's taxonomy [17] offers a valuable framework for classifying the diverse manifestations of natural background noise. The delineation into three primary categories—microseism peak, short-period noise, and long-period noise—provides a structured approach to understanding the myriad phenomena that contribute to the ambient acoustic landscape [18]. From the rhythmic undulations of Earth tides to the ethereal whispers of low-frequency waves, from the turbulent symphony of atmospheric pressure changes to the gusts of winds and currents, and from the subtle tilting motions to the reverberations of seismic waves traversing the Earth's crust, each category encapsulates a kaleidoscope of geological dynamics spanning different frequency bands and geological contexts.

As geological engineering endeavors continue to evolve, this expanded comprehension of noise's intricate role promises to enrich our understanding of Earth's dynamic processes, empowering us to better predict and mitigate the impacts of natural hazards while harnessing the Earth's resources sustainably.

The term $(au(t) - bf(u(t)))$ represents a general expression for the nonlinear friction term (Fig. 3).

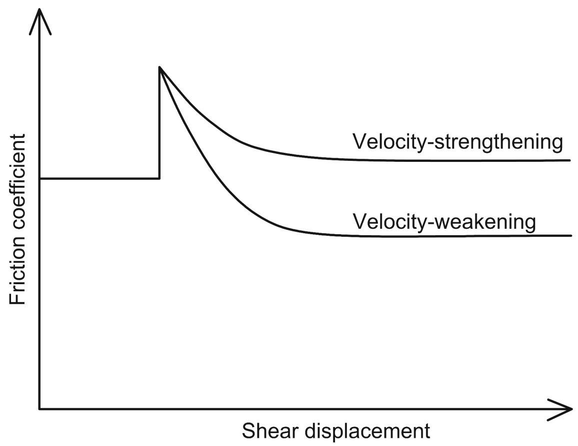

Fig. 3. Nonlinear friction laws for the earthquake fault motion, are generally considered in the dynamical modeling of fault motion.

Various rate/state-variable constitutive laws have been employed to replicate laboratory observations of rock friction, as documented by Scholz [19]. Among these, the Dieterich-Ruina law, also known as the "slowness" law, currently stands out for its remarkable alignment with experimental findings. Figure 3 provides a schematic yet accurate representation of this law, illustrating the observed frictional response to a sudden e-fold increase followed by a subsequent decrease in sliding velocity. This law elucidates two key phenomena: the direct velocity effect, where friction increases upon the initial application of the rate increase, followed by an evolutionary effect leading to a reduction in friction.

Central to this framework is the concept of creep in surface contact, which gradually augments the real contact area over time, thereby giving rise to associated behaviors such as aging and velocity dependence. The critical slip distance serves as a memory distance governing the variation in the contact population. As an empirical and heuristic model, the rate/state-variable constitutive law adeptly captures the full spectrum of experimental data, demonstrating its applicability across diverse materials ranging from paper and wood to certain metals. Notably, this model eliminates the traditional differentiation between static and dynamic friction coefficients.

The influence of temperature and rock type on base friction is minimal, indicating that the fundamental frictional behavior remains largely unaffected by these factors. Rather, the intriguing behaviors under discussion stem from second-order effects that hinge on both a state variable and sliding velocity. Importantly, this discourse pertains not to base friction, which establishes the fault's inherent strength, but rather to frictional stability, which governs the fault's propensity for seismic activity. Hence, while strength delineates the fault's resistance to sliding, it is the fault's stability that ultimately dictates its seismic behavior.

Now we require that the functions f and g satisfy the usual uniform Lipschitz condition, i.e.

$$\left|bf(u_i(t)) + cg\left(u_i^{\tau_1}(t)\right) - bf\left(u_j(t)\right) + cg\left(u_j^{\tau_1}(t)\right)\right| \leq k(|u_i(t) - u(t)| + \left|u_i^{\tau_1}(t) - u_J^{\tau_1}(t)\right|) \tag{2}$$

Let $\Delta_i(t) = u(t) - u_{i-1}(t)$, $i = 2,3,\ldots,N,\ldots$ denote the difference (synchronization error) between the solution $u_i(t)$ and the nearest neighbor solution $u_{i-1}(t)$, where $i = 2,3,\ldots,N$. Then, $d\Delta_i(t)$ can be estimated using the Lipschitz condition as follows

$$d\Delta_i(t) \leq (-a\Delta_i(t) + k|\Delta_i(t)| + k|\Delta_i(t-\tau_1)| - (d_1+d_2)\Delta_i^{\tau_2}(t) + d_2\Delta_{i+1}^{\tau_2} + d_1\Delta_{i-1}^{\tau_2}(t))dt + \Delta_i(t)\sqrt{2D}dW \tag{3}$$

where $\Delta_i(t-\tau) = u_i(t-\tau) - u_{i-1}(t-\tau) \equiv \Delta_i^{\tau}(t)$ and $i = 3,4,\ldots,N-1$.

3 Results

If constants *a, k, d_1, d_2* and *D* satisfy.

$$k + d_1 + d_2 < 0.5(a - -D) \tag{4}$$

then the system (3) is exponentially stable in the mean square.

Applying the generalized Ito formula to $\sum_{i=2}^{N} \Delta_i^2(t)$, we have

$$d\left(\sum_{i=2}^{N} \Delta_i^2(t)\right) \le \sum_{i=2}^{N}\Big[-2a\Delta_i^2(t) + 2k\Delta_i(t)|\Delta_i(t)| + 2k\Delta_i(t)|\Delta_i(t-\tau_1)| - 2(d_1+d_2)\Delta_i(t)\Delta\Delta_i^{\tau_2}(t))dt + 2\Delta_i^2(t)\sqrt{2D}dW(s)\Big] + \sum_{i=3}^{N} 2d_1\Delta_i(t)\Delta_{i-1}^2(t)dt + \sum_{i=2}^{N-1} 2d_2\Delta_i(t)\Delta_{i+1}^2(t)dt + \sum_{i=2}^{N} \Delta_i^2(t)2Ddt \tag{5}$$

where k, d_1, d_2, a, $D \ge 0$. For $U = \left(\Delta_2^2(s), \ldots, \Delta_N^2(s), s\right) = \sum_{i=2}^{N} e^{-2\alpha(t-s)}\Delta_i^2(s)$, and integrating with respect to s, we get $\int_0^t d\left[e^{-2\alpha(t-s)}\Delta_i^2(s)\right] = \int_0^t \sum_{i=2}^{N}\left[\frac{\partial U}{\partial s}ds + \frac{\partial U}{\partial x_i}dX_i + \frac{1}{2}\frac{\partial^2 U}{\partial x_i^2}(G_i)^2 ds\right]$, where $X_i = \Delta_i(s)$ and $dX_i = d\Delta_i(s) = F_i ds + G_i dW(s)$.

Thus,

$$\sum_{i=2}^{N} \Delta_i^2(t) \le e^{-2at}\sum_{i=2}^{N} \Delta_i^2(0) + \int_0^t e^{-2a(t-s)}\Big\{\sum_{i=2}^{N}\Big[2k\Delta_i^2(s) + 2k|\Delta_i(s)||\Delta_i(s-\tau_1)| + 2(d_1+d_2)|\Delta_i(s)|\left|\Delta_i^{\tau_2}(s)\right|ds + 2\sqrt{2D}\Delta_i^2(s)dW(s) + 2D\Delta_i^2(s)ds\Big] + \sum_{i=3}^{N} 2d_1|\Delta_i(s)|\left|\Delta_{i-1}^{\tau_2}(s)\right|ds + \sum_{i=2}^{N-1} 2d_2|\Delta_i(s)|\left|\Delta_{i+1}^{\tau_2}(s)\right|ds\Big\} \tag{6}$$

We state there exists some sufficiently small positive constant λ, $a > \lambda > 0$, such that

$$a - \lambda - k - ke^{\lambda t} - 2(d_1 + d_2)e^{\lambda t} - D > 0 \tag{7}$$

where $\tau = max(\tau_1, \tau_2)$.

Let us denote by $G(t) = \sup_{\substack{-\tau \le \theta \le t \\ -\tau \le \psi \le t \\ 2 \le i,j \le N}} E\left[|\Delta_i(\theta)|\left|\Delta_j(\psi)\right|\right]e^{\lambda\theta}e^{\lambda\psi}$. Then using

$E\left(\int_0^t e^{[2\lambda t-2a(t-s)]}2\sqrt{2D}\sum_{i=2}^{N}\Delta_i^2(s)dW(s)\right)=0$,we have

$$E\left[\sum_{i=2}^{N}\Delta_i^2(t)\right]e^{2\lambda t}\le e^{[2\lambda-2a]t}E\left[\sum_{i=2}^{N}\Delta_i^2(0)\right]+ \int_0^t e^{[2\lambda t-2a(t-s)]}\left\{2k\sum_{i=2}^{N}E\left[\Delta_i^2(s)\right]e^{2\lambda s}e^{-2\lambda s}+2k\sum_{i=2}^{N}E[|\Delta_i(s)||\Delta_i(s- \tau_1)|]e^{[\lambda s]}e^{[\lambda(s-\tau_1)]}e^{-\lambda s}e^{[-\lambda(s-\tau_1)]}+2(d_1+d_2)\sum_{i=2}^{N}E[|\Delta_i(s)||\Delta_i(s- \tau_2|]e^{[\lambda s]}e^{[\lambda(s-\tau_2)]}e^{-\lambda s}e^{[-\lambda(s-\tau_2)]}+2D\sum_{i=2}^{N}E\left[\Delta_i^2(s)\right]e^{2\lambda s}e^{-2\lambda s}+ 2d_1\sum_{i=3}^{N}E\left[|\Delta_i(s)||\Delta_{i-1}(s-\tau_2|\right]e^{[\lambda s]}e^{[\lambda(s-\tau_2)]}e^{-\lambda s}e^{[-\lambda(s-\tau_2)]}+ 2d_2\sum_{i=2}^{N-1}E\left[|\Delta_i(s)||\Delta_{i+1}(s-\tau_2|\right]e^{[\lambda s]}e^{[\lambda(s-\tau_2)]}e^{-\lambda s}e^{[-\lambda(s-\tau_2)]}\right\}ds, \tag{8}$$

Thus, we obtain

$$E\left[\sum_{i=2}^{N}\Delta_i^2(t)\right]e^{2\lambda t}\le e^{[2\lambda-2\alpha]t}E\left[\sum_{i=2}^{N}\Delta_i^2(0)\right]+\int_0^t e^{[2\lambda-2\alpha](t-s)}ds[2k(N- 1)G(t)+2ke^{\lambda t}(N-1)G(t)+2(d_1+d_2)e^{\lambda t}(N-1)G(t)+2D(N-1)G(t)+ 2d_1e^{\lambda t}(N-2)G(t)+2d_2e^{\lambda t}(N-2)G(t)\Big],$$
$$E\left[\sum_{i=2}^{N}\Delta_i^2(t)\right]e^{2\lambda t}\le E\left[\sum_{i=2}^{N}\Delta_i^2(0)\right]+\frac{1}{[2\alpha-2\lambda]}\Big[\big(2k+2ke^{\lambda\tau}+2(d_1+d_2)e^{\lambda\tau}+ 2D\big)(N-1)G(t)+2(d_1+d_2)e^{\lambda\tau}(N-2)G(t)\Big]. \tag{9}$$

Now, we get

$$[2a-2\lambda](N-1)G(t)\le[2a-2\lambda]E\left[\sum_{i=2}^{N}\Delta_i^2(0)\right]+\Big[\big(2k+2ke^{\lambda\tau}+2(f_1+ d_2)e^{\lambda\tau}+2D\big)(N-1)G(t)+2(d_1+d_2)e^{\lambda\tau}(N-2)G(t)\Big],$$
$$\big[2a-2\lambda-2k-2ke^{\lambda t}-4(d_1+d_2)e^{\lambda t}-2D\big](N-1)G(t) \le[2a-2\lambda]E\left[\sum_{i=2}^{N}\Delta_i^2(0)\right], \tag{10}$$

which finally gives

$$E\left[\sum_{i=2}^{N}\Delta_i^2(t)\right]\le\frac{2aE\left[\sum_{i=2}^{N}\Delta_i^2(0)\right]e^{-2\lambda t}}{[2a-2\lambda-2k-2ke^{\lambda t}-4(d_1+d_2)e^{\lambda t}-2D]}. \tag{11}$$

4 Numerical Example

We demonstrate the validity of our theorem for the 96-unit stochastic earthquake fault motion model with the delayed coupling of general form (2), where functions f(v) and g(v) have the corresponding general forms of arctan (t) and sin (t). The inverse trigonometric function, also known as normally consolidated clays, can be viewed as a qualitative representation of the behavior of plastic soil. Lipschitz conditions are met by both functions ($k = 1$).

In the numerical example, for the sake of simplicity, we took the same parameter values $\beta = \gamma = c1 = c2 = 1$. Noise intensity is assumed to be low ($D = 0.1$), while values for delay terms are chosen arbitrarily: $\tau 1 = 0.23$, $\tau 2 = 0.42$. The purpose of the numerical example is to illustrate the synchronization when derived synchronization conditions are met. If the theorem's conditions are not met, we used $\alpha = 1$. Our theorem states that for the theorem's conditions to be satisfied, α must be greater than 6.1. Figure 1a depicts this instance. The computation results when the theorem conditions are met ($\alpha = 6.2$) are displayed in Fig. 1b. Evidently, when the conditions of the theorem are met, synchronization of units is observed (Fig.4). AQ3

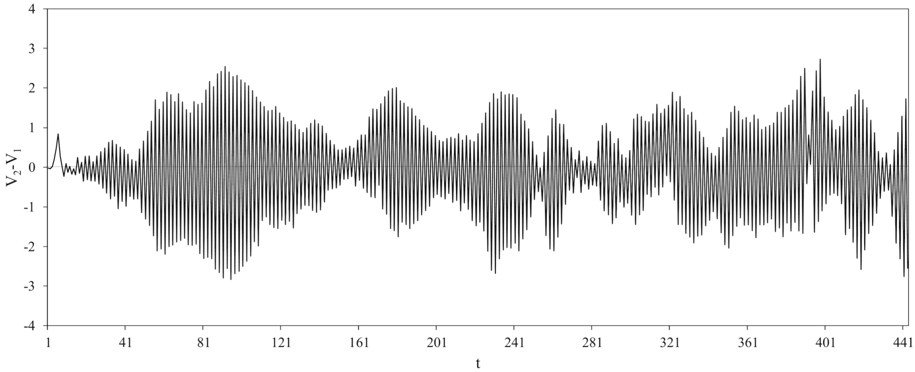

Fig. 4. A numerical example, illustrating the synchronized motion of the model whose dynamics are described by Eq. (1). Parameter values for the model are $\beta = \gamma = c1 = c2 = 1$, $D = 0.1$, $\tau 1 = 0.23$, $\tau 2 = 0.42$.

5 Conclusion

In our innovative exploration of one-dimensional earthquake dynamics, we have embarked on a thorough examination of the stochastic stability inherent in achieving precise synchronization. Our model operates within the realm of block velocities, where the subtle differences between a block's velocity and those of its immediate neighboring blocks, both ahead and behind, serve as the determinants of each block's acceleration. What distinguishes our model is its meticulous consideration of various forms of feedback delays and the influence of noise, factors that significantly shape the dynamics of seismic events.

At the heart of our mathematical framework lies an ensemble of N scalar nonlinear stochastic Delay Differential Equations (DDEs), meticulously crafted to capture the intricacies of seismic interactions. Each unit within this interconnected chain is characterized by nonlinear functions, carefully crafted to be piece-wise continuous while adhering to the uniform Lipschitz condition. Despite this adherence, the internal parameters and feedback delays imbue each unit's dynamics with inherent complexity, rendering their behavior notably intricate when considered in isolation from the larger system.

Our investigation has yielded a profound insight into the conditions conducive to achieving exact synchronization in the mean square. Through rigorous analysis, we have identified a clear and sufficient condition hinging on the coupling constants (c_2, c_1) and the Lipschitz constant (k). Remarkably, these parameters emerge as both necessary and sufficient for attaining global asymptotic synchronization in the mean square. Furthermore, we have elucidated that this critical condition can be approximated through the manipulation of feedback delays (τ_1) and internal parameters, remaining independent of both the coupling delay (τ_2) and the total number of units (N).

Translating our findings into practical terms unveils their profound implications for seismic hazard assessment and mitigation strategies. By discerning the prerequisites for synchronization among different fault segments or adjacent faults, our research offers a pathway to better understanding seismic phenomena's complex dynamics. Moreover, the aggregation of numerous active individual units underscores the potential for their combined effect to yield significantly amplified deformations, thereby highlighting the importance of our findings in the realm of seismic risk management and disaster preparedness.

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Author Proof

Author Queries

Chapter 54

Query Refs.	Details Required	Author's response
AQ1	Please check whether the following orcid id [0000-1111-2222-3333] and [1111-2222-3333-4444] seems invalid. Kindly provide valid one.	Srdjan Kostic [0000-0002-3705-3080] Nebojsa Vasovic [0000-0002-8294-5117]
AQ2	The citation of "Fig. 3" has been modified as "Fig. 2" in Paragraph starting with "Regarding the noise effect...". Please check, and correct if necessary.	Checked, correct.
AQ3	Please check and confirm if the inserted citation of Fig. 4 is correct. If not, please suggest an alternate citation.	Checked, correct.