

Analysis of kinetics of local hydraulic losses on the laterals of radial wells at Belgrade groundwater source

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ABSTRACT

An analysis of the dynamics of the process of biochemical colmation of laterals of radial wells at the Belgrade groundwater source is presented. A very good correlation between the rate of forming of the colmations and the concentration of bivalent iron was obtained, on the basis of which the maximum recommended flows and velocities were defined, which are in good agreement with the values from previous articles and studies. The effects of regenerations on yield and the total volume of extracted water (up to 2 times higher than in the case of no regeneration) were investigated using a mathematical and software model. A correlation was developed linking the ratio of the amounts of water extracted with and without regeneration, to the aquifer hydraulic resistance, the coefficient of local hydraulic losses reduction due to regeneration, and the number of regenerations. The factor equal to the square root of the number of regenerations increased by one was added to the expressions for the maximum recommended values of inlet velocities and flows per lateral. The differences in flows and extracted volumes between operating modes with constant flow between regenerations and a constant, minimal water level were also examined using the model. The ratio between the total extracted volumes when the drawdown is kept equal to maximum and when the flow between the regenerations is kept constant reaches up to 1.25.

Key words: dissolved iron, laterals, local hydraulic losses, radial wells, regeneration

HIGHLIGHTS

- Accurate correlation between the bivalent iron concentration in groundwater (and flow per lateral) and rate of increase of local hydraulic losses.
- Mathematical and software model for the calculation of drawdown in radial wells that includes the effects and number of regenerations and rate of increase of the local hydraulic losses for the constant flow or constant level between the regenerations modi operandi.

INTRODUCTION

Reduction of the yield of Belgrade groundwater source (BGS) radial wells is caused by the decrease in the area through which the water flows, due to clogging and narrowing of the pore space in the zone near the laterals and a decrease in the number and length of active laterals. This is due to two dominant causes: microbial colmation and corrosion (Dimkić *et al.* 2011a). The alluvial groundwaters are a valuable resource, as, for example, many pharmaceuticals and other organic compounds are eliminated at BGS through bank and aquifer filtration processes (Kovačević 2017; Kovačević *et al.* 2017), which eliminates the need for sophisticated treatment methods (for example Zinatloo-Ajabshir *et al.* 2020, 2021). The processes causing the reduction in yield, their correlation with selected parameters of groundwater quality, theoretical basis for defining the parameters that determine the hydraulic performance of the well and its temporal change, are presented in Dimkić *et al.* (2011a, 2011c) and Institute Jaroslav Černi (2010). Regression functions correlating the rate of increase of hydraulic losses to dissolved bivalent iron concentration and redox potential of groundwater are presented in Dimkić & Pušić (2014) and Institute Jaroslav Černi (2010).

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Reduction of the number of laterals is a significant consequence of regenerations. The laterals are sealed off when they are physically damaged to the point when the aquifer material breaks into the interior of the lateral and closes it, often near the well shaft, which results in a drastic decrease in flow and large increase in the content of sand in the water.

In case that the material of which the laterals are made is prone to corrosion, and groundwater conditions are favorable, corrosion first causes an increase in the roughness of the material, resulting in large areas becoming more suitable for the development of microorganisms and the accumulation of their extracellular products and mineral deposits. This in turn causes the formation of numerous micro zones with conditions even more favorable for the corrosion, as well as the clogging of the pores in the zone around the laterals. The deterioration of the material of the laterals, combined with strong mechanical shocks during regenerations by jetting, eventually causes their destruction. Less than one third of all laterals has retained the structural integrity along the entire original length (Polomčić *et al.* 2016).

If the corrosion processes are not intense, the decrease in flow through the laterals which mechanical integrity is preserved, is caused by an increase in local resistances due to the clogging of pore space by accumulation of micro-organism products and their biomass. The dynamics of these processes can therefore be studied best by observing wells with laterals made of corrosion-resistant stainless steel. All laterals installed before 2005 at BGS were made of ordinary carbon steel. From that year on, stainless steel laterals were installed (using Preussag method) (Dimkić *et al.* 2011a). Since there are only few such wells, at present 8 (Rb-1, Rb-5 m, Rb-8 m, Rb-8, Rb-15, Rb-16, Rb-18 and Rb-20), out of which for five of them we have data (Rb-5 m, Rb-8, Rb-15, Rb-16 and Rb-20), some wells with older laterals were also selected to investigate the effects and causes of colmation (those for which it was assumed that the colmation dominated, and for which there are more than three data on local hydraulic losses during the period between 2006 and 2013). The number of active laterals, six or more active laterals in 2013, was used as a criterion by which the wells with dominant biochemical clogging were separated from other wells, a total of 29 wells.

Effects of well regenerations on hydraulic losses were quantitatively evaluated, and 41 periods of operation uninterrupted by regenerations were singled out and average values of flow per lateral, dissolved bivalent iron concentration and redox potential calculated for each period. The objectives were: (1) to obtain the best possible correlations between these parameters and the rates of colmation, (2) to develop a new mathematical model for calculation of drawdown in wells that includes the effects of regenerations, and the calculation of the increase of local hydraulic losses using the best of aforementioned correlations, and (3) to use this model as a base for simulation model implemented using Excel and macros in Visual Basic for Applications. The main result obtained by analyses using the simulation model was the quantification of impact of regenerations, and of the difference in the total amount of extracted water between the well operation at constant flow and at constant drawdown between the regenerations.

METHODS

Local hydraulic losses – definition, determination, and correlation with flow, redox potential and concentration of dissolved bivalent iron

Figure 1(a) shows a typical flow pattern around the single well with horizontal laterals in the vertical section.

In order to estimate the magnitude of the local resistance on the laterals, as well as the effects of the regenerations performed, it is necessary to quantify the local hydraulic resistance (*LHR*) of the laterals and the near-lateral zone (Dimkić *et al.* 2008, 2012). *LHR* is defined as the ratio between the difference in piezometer levels between the well shaft and the nearby piezometer located in the immediate zone of lateral (ΔS), and the flow per lateral (q) (which is in this paper expressed in liters per second) (Equation (1)).

$$\text{LHR} = \frac{\Delta S}{q} \left[\frac{\text{m}}{\text{l/s}} \right] \quad (1)$$

ΔS is the difference between the levels in the near piezometer and the well, and consists of drawdown due to the flow through the aquifer – S' , and drawdown (variable even for constant flow) resulting from hydraulic resistances near, and on the lateral – S'' (Dimkić *et al.* 2011c), Figure 1(b). The resistance of the aquifer x is given by Equation (2) (drawdown per unit of flow per lateral):

$$x = \frac{S + S'}{q} \quad (2)$$

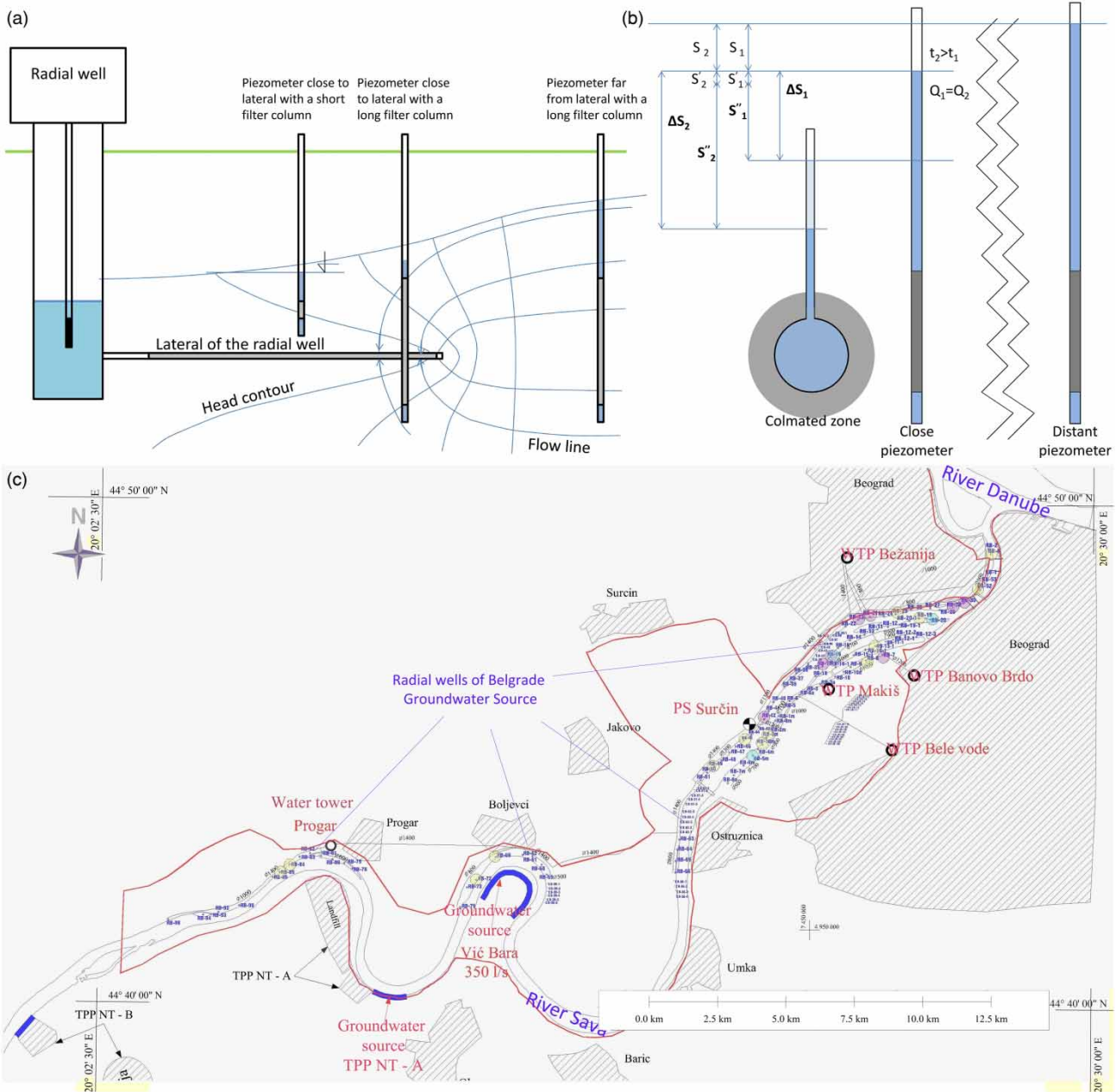


Figure 1 | (a) Typical flow pattern around the well with horizontal laterals (vertical section through the lateral) – Mitrinović *et al.* (2018). (b) Loss of groundwater head from the far field to the point near the lateral, through the colimated zone, and into the interior of the lateral, at constant flow conditions (Mitrinović *et al.* 2018). (c) Belgrade groundwater source system (taken with modifications from Jaroslav Černi Institute (2010)).

The maximum available drawdown ΔS_{max} is constrained by the static piezometer level and the minimum pump suction depth below the water level in the well shaft.

If the characteristics of the aquifer material released does not change outside the immediate zone of laterals, the following applies (Dimkić *et al.* 2011c):

$$\frac{\partial}{\partial t} \left(\frac{S'}{q} \right) \approx 0 \tag{3}$$

It is possible to use piezometers outside the zone of laterals, in which case a significant part of the difference between the levels in the piezometer and the well shaft is a depression due to the flow through the aquifer. The *LHR* values in this case are overestimated. When observing a period without regeneration, the *LHR* almost always increases linearly with time when the flow in the well does not change drastically. Rate of change of local hydraulic resistance, i.e. their kinetics, is called *KLHR* (kinetics of local hydraulic resistance) (Dimkić *et al.* 2011a, 2011c), and usually does not change much over time, i.e. it is equal to the slope of the linear regression function for *LHR*, and is expressed as:

$$KLHR = \frac{\partial}{\partial t} \left(\frac{\Delta S}{q} \right) = \frac{\partial}{\partial t} \left(\frac{S''}{q} \right) \quad (4)$$

The flow through the lateral (Equation (5)) and the inlet velocity (Equation (6)), for which, without change in flow and velocity, all available drawdown will be used at the end of the observed period of length t , can be determined through *KLHR*, based on the maximum available drawdown and the resistance of the aquifer x :

$$\frac{\Delta S_{\max} - x \cdot q_0}{KLHR_0 \cdot t} = \frac{S''_{(t)}}{KLHR_0 \cdot t} = q_0 \quad (5)$$

$$\frac{\Delta S_{\max} - x \cdot q_0}{KLHR_0 \cdot t} \cdot \frac{1}{1,000 \cdot d \cdot \pi \cdot l} = \frac{S''_{(t)}}{KLHR_0 \cdot t} \cdot \frac{1}{1,000 \cdot d \cdot \pi \cdot l} = v_0 \quad (6)$$

In Equation (6), l and d are the length and diameter of the perforated part of the lateral expressed in meters, respectively. Alternatively, the maximum increase of hydraulic losses on laterals during one year can be set as $\Delta S_{\max a}$ (a for annual), so the corresponding flow per lateral and inlet velocity are given by Equations (7) and (8).

$$\frac{\Delta S_{\max a}}{KLHR_0} = q_{\max} \quad (7)$$

$$\frac{\Delta S_{\max a}}{KLHR_0} \cdot \frac{1}{1,000 \cdot d \cdot \pi \cdot l} = v_{\max} \quad (8)$$

Effects of regeneration of laterals

In the case when regeneration is performed, the change in *LHR* is practically instantaneous, and most often negative. For a realistic assessment of the effects of the regenerations, it is necessary to quantify the *LHR* value before and after. The coefficient of reduction of local hydraulic losses due to regeneration y_{reg} is given as:

$$y_{reg} = \frac{LHR_2}{LHR_1} \quad (9)$$

where LHR_1 and LHR_2 are *LHR* values immediately before and after regeneration, respectively, which can be determined by extrapolation of regression functions before and after regeneration (Figure 2), forward and backward, to the moment of regeneration.

Location, period and method of monitoring and analysis of work and performance of wells

From 2006 to 2013, the flows in wells with horizontal laterals, groundwater levels in near piezometers, and the ones in the hinterland and between the wells, changes due to regeneration, as well as the physico-chemical quality of water (in the laboratory and *in situ*) were systematically monitored at BGS (Jaroslav Černi Institute 2010). Campaigns of well flow measuring and *LHR* values determining were conducted approximately two to three times a year, using a method based on a brief shut-down of the pumps (about 15 min) and monitoring the inflow of water into the well. This method was chosen as the wells were not evenly equipped with flow meters, and this assured that the measurements would be carried out in the same way at each well, with the satisfactory level of measurement accuracy. When measuring the capacity of wells, the parts of the groundwater source in which the measurements were performed were previously brought into a quasi-stationary state of exploitation by neighboring wells working continuously for at least 48 h. The level in the well was measured and recorded

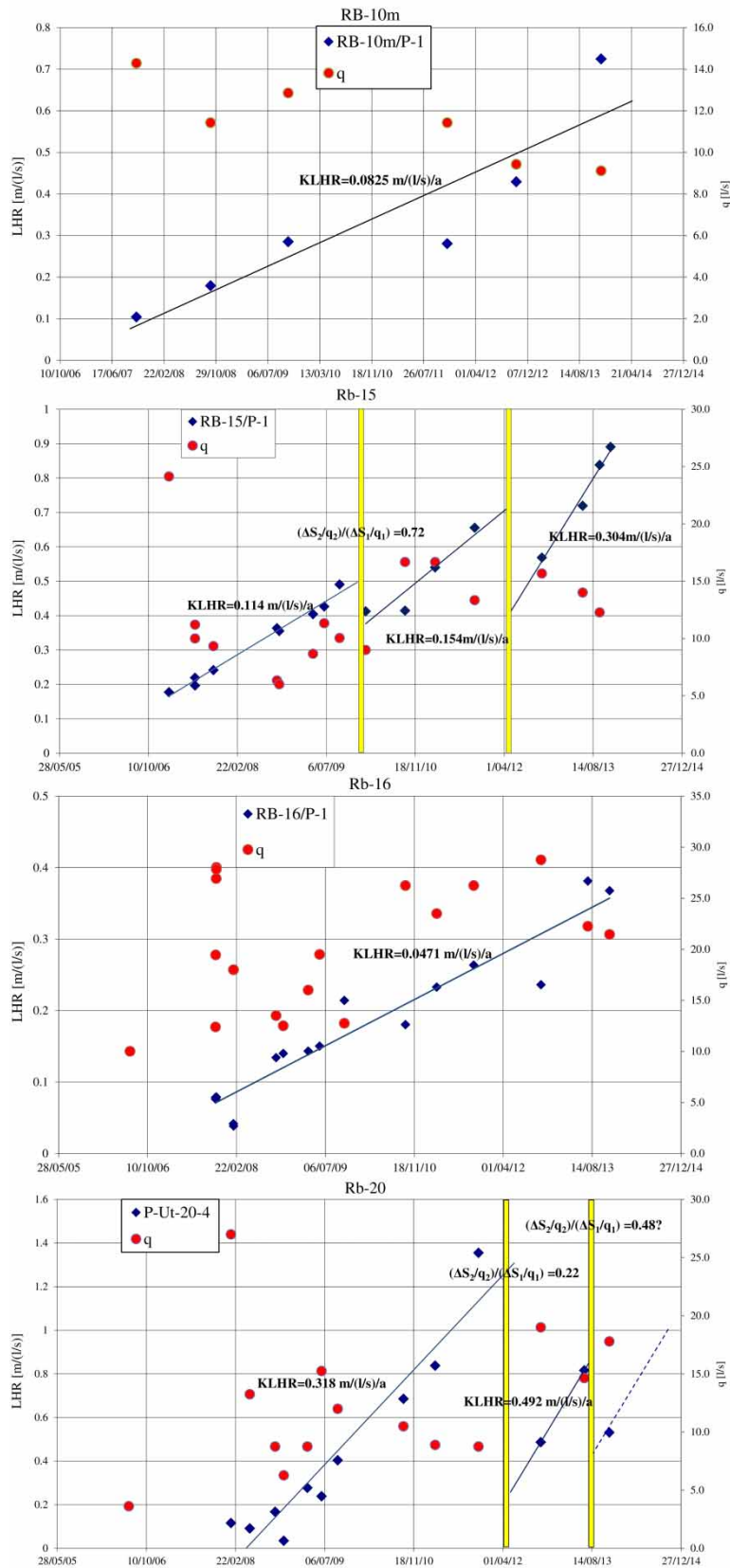


Figure 2 | Temporal change of *LHR*, *KLHR* and flows per lateral for Rb-10 m (piezometer Rb-10 m/p-1), Rb-15 (piezometer Rb-15/p-1), Rb-16 (piezometer Rb-16/p-1) and Rb-20 (piezometer P-Ut-20-4). Yellow lines represent regenerations.

every second using logging divers, and at the same time monitored using a manual level meter. Based on the diameter of the well and the change in the water level in the well, the volume of water that entered the well in a time interval was obtained, which represents the value of the flow of a given well. Changes in groundwater levels in close piezometers were also monitored, and local drawdown ΔS calculated as the differences in levels between the well and the piezometer (Jaroslav Černi Institute 2010). In addition to hydraulic measurements, *in situ* measurements of physical and chemical parameters of water quality (temperature, standard hydrogen electrode redox potential, specific electrical conductivity, dissolved oxygen content, *pH*) were performed using probes. After measuring the physico-chemical characteristics, water was sampled from a well and a nearby piezometer for laboratory physico-chemical analyses of groundwater. Significant volume of data on the condition of the well and its change over time was obtained, as well as on the influence of physico-chemical parameters of groundwater quality on these changes.

Investigation of correlations of KLHR with flow, redox potential and dissolved ferrous iron concentration, analysis of well operation and performance

The periods over which all 29 selected wells were monitored were divided into the sub-periods between regenerations, if there were any. Linear regression of *LHR* values was performed for each of the periods to obtain *KLHR*, and average values of flow per lateral, dissolved bivalent iron concentration and redox potential were calculated. Periods for which there was insufficient data were not taken into consideration. In this way, 41 periods with the corresponding average values of flow per lateral, bivalent iron concentration, redox potential and *KLHR* were isolated.

Previous research has shown that there is a relationship between *KLHR* and dissolved bivalent iron concentration, *KLHR* and flow per lateral or inlet water velocity, as well as *KLHR* and groundwater redox potential (Dimkić *et al.* 2008; Jaroslav Černi Institute 2010; Dimkić *et al.* 2011c; Dimkić & Pušić 2014). Therefore, the correlations between *KLHR* and average values of flow per lateral, dissolved bivalent iron concentration and redox potential for all wells, and all 41 periods of their operation, were examined. Functions of different types were examined, of which the best regression results were obtained for combinations of power and exponential function for the correlation of *KLHR* with the flow per lateral and concentration of dissolved bivalent iron, or *KLHR* with flow per lateral and redox potential. Correlation procedure was performed using Excel Solver in such a way that the values of the parameters in the correlation function were automatically varied so that a minimum value was obtained in the target cell, which contained the mean squared error.

Examination of correlations of regeneration effects with potentially relevant factors

Evaluation of regeneration results included testing the correlation of y_{reg} with dissolved bivalent iron concentration, redox potential, number of regenerations performed so far on the well, *LHR* value before regeneration, number of active laterals before regeneration and flow per lateral, using basic forms of functions available within trend line option in Microsoft Excel.

Mathematical model for the calculation of drawdown

As established in previous research (Dimkić & Pušić 2008; Jaroslav Černi Institute 2010; Dimkić *et al.* 2012; Dimkić & Pušić 2014), as well as in the analysis presented in this paper (Equation (24) in the Results section), *KLHR* is proportional to the flow through the lateral, so *KLHR* in all subsequent periods can be presented as a product of *KLHR* value in the first period ($KLHR_0$) and ratios between flow per lateral in the given period and in the first period (provided that the concentration of dissolved bivalent iron is approximately constant), where y_n is the ratio between flows per lateral (q_i) in successive periods (n) between regenerations (Equations (10) and (11)).

$$y_n = \frac{q_{n+1}}{q_n} \quad (10)$$

$$KLHR_n = KLHR_0 \frac{q_n}{q_0} = KLHR_0 \prod_{i=0}^{n-1} y_i \quad (11)$$

$KLHR_0$ is the value of the kinetics of local hydraulic losses in case there is no regeneration and for the flow q_0 . It then corresponds to some value of the concentration of bivalent iron according to Equation (24). The *KLHR* in the first period, for

example, is calculated as:

$$KLHR_1 = KLHR_0 y_0 \quad (12)$$

when defining the life span of laterals t or simply the time period considered, and the number of regenerations n , the draw-down before regeneration at the end of the period can be calculated using equations such as (13) to (18), for the first, second, third, fourth and fifth periods (for the case of four regenerations), as well as for the general case, respectively:

$$\Delta S_{\max} = (x + LHR_1) \cdot q_1 = \left(x + KLHR_1 \frac{t}{5}\right) \cdot q_1 = \left(x + KLHR_0 y_0 \frac{t}{5}\right) \cdot q_0 y_0 \quad (13)$$

$$\Delta S_{\max} = \left(LHR_1 y_{\text{reg}} + x + KLHR_2 \frac{t}{5}\right) \cdot q_1 y_1 = \left(KLHR_0 y_0 \frac{t}{5} y_{\text{reg}} + x + KLHR_0 y_0 y_1 \frac{t}{5}\right) \cdot q_0 y_0 y_1 \quad (14)$$

$$\begin{aligned} \Delta S_{\max} &= \left(LHR_2 y_{\text{reg}} + x + KLHR_3 \frac{t}{5}\right) \cdot q_1 y_1 y_2 = \\ &= \left(\left(KLHR_0 y_0 \frac{t}{5} y_{\text{reg}} + KLHR_0 y_0 y_1 \frac{t}{5}\right) \cdot y_{\text{reg}} + x + KLHR_0 y_0 y_1 y_2 \frac{t}{5}\right) \cdot q_0 y_0 y_1 y_2 \end{aligned} \quad (15)$$

$$\begin{aligned} \Delta S_{\max} &= \left(LHR_3 y_{\text{reg}} + x + KLHR_4 \frac{t}{5}\right) \cdot q_1 y_1 y_2 y_3 = \\ &= \left(KLHR_0 y_0 \frac{t}{5} y_{\text{reg}}^3 + KLHR_0 y_0 \frac{t}{5} y_{\text{reg}}^2 y_1 + \right. \\ &\quad \left. + KLHR_0 y_0 \frac{t}{5} y_{\text{reg}} y_1 y_2 + x + KLHR_0 y_0 \frac{t}{5} y_1 y_2 y_3\right) \cdot q_0 y_0 y_1 y_2 y_3 \end{aligned} \quad (16)$$

$$\begin{aligned} \Delta S_{\max} &= \left(LHR_4 y_{\text{reg}} + x + KLHR_5 \frac{t}{5}\right) \cdot q_1 y_1 y_2 y_3 y_4 = \\ &= \left(KLHR_0 y_0 \frac{t}{5} y_{\text{reg}}^4 + KLHR_0 \frac{t}{5} y_{\text{reg}}^3 y_0 + KLHR_0 y_0 \frac{t}{5} y_{\text{reg}}^2 y_1 y_2 + \right. \\ &\quad \left. + KLHR_0 y_0 \frac{t}{5} y_{\text{reg}} y_1 y_2 y_3 + x + KLHR_0 y_0 y_1 y_2 y_3 y_4 \frac{t}{5}\right) \cdot q_0 y_0 y_1 y_2 y_3 y_4 \end{aligned} \quad (17)$$

$$\Delta S_{\max} = KLHR_0 \left(\sum_{i=0}^n \left(\frac{t}{n+1} y_{\text{reg}}^i \prod_{j=0}^{n-i} y_j \right) + \frac{x}{KLHR_0} \right) \cdot q_0 \prod_{k=0}^n y_k \quad (18)$$

Software model of well operation based on mathematical model for the calculation of drawdown

Constant flow in between the regenerations

The LHR at the end of each period is calculated as the sum of the resistance of the aquifer x , local hydraulic losses resulting from incrustations from previous periods (obtained by calculating the gradual growth of LHR depending on the corresponding $KLHR$ in each previous period, and applying the coefficient of reduction of local hydraulic losses due to regeneration (y_{reg}) for all previous regenerations), and local hydraulic losses incurred during the observed period. Drawdown at the end of that period is obtained by multiplying LHR with the flow per lateral during the observed period (Equations (13)–(18)).

The maximum available drawdown ΔS_{\max} is set by the static piezometric level and the minimum pump suction depth below the level in the well shaft. Maximum flows and maximum total amount of pumped water during each period are achieved by utilizing the total available drawdown, so it is assumed that the value of ΔS_{\max} for each period is the same and equal to total available drawdown, assuming that the general goal is to use the well capacity as much as possible. The flow per lateral in between the regenerations is constant.

Equations (13)–(18) were solved for 11 values of the coefficient of reduction of local hydraulic losses due to regeneration y_{reg} (from 0 to 1 in steps of 0.1), to obtain values from y_0 to y_{n-1} for each y_{reg} , which was carried out using Excel Solver. The value of ΔS_{\max} is subtracted from the value of the calculated drawdown for the end of each period, and the absolute values of

the obtained results for all y_{regs} are summed in the target cell. The values of the cells containing y_n s are automatically varied using Excel Solver, until the value of the target cell reaches minimum, i.e. value closest to zero. According to Equation (19), the value of q_0 is obtained from $KLHR_0$:

$$q_0 = \frac{\Delta S_{\max}}{KLHR_0 \cdot t + x} \quad (19)$$

where t is the length of the observed time period, and the flows per lateral are obtained from q_0 for each sub-period separated by regenerations (q_1 to q_n) by using y_0 to y_{n-1} , and from them the drawdown in the well is calculated according to Equations (13)–(18).

Constant level in between the regenerations

In practice, the operation of the well pump is usually regulated in such a way as to keep the level constant, hence the flow changes over time. Using the presented method, this case can be simulated by dividing the periods between regenerations (or, if there are none, the whole observed period) into a large number of intermediate periods within which the level does not change much even when the flows per lateral are large. In Excel Solver, it is not possible to vary the value of a sufficiently large number of variables (in this approach to the problem), in this case the value of y_n , i.e. the ratio between the flows per lateral in successive intermediate periods, and in addition the macro within which Solver would be used could only be used for a predefined number of periods. Also, the variation of y_n affects the calculated value of level drop through Equation (18), and it is hardly feasible for it to be developed in a single spreadsheet cell for a large number of periods, even by combining several of them.

Due to the mentioned limitations, alternative, less precise method using a macro was applied instead of the Solver – q_n was changed in steps of 0.01 starting from the value of 0.01 L/s and the drawdown at the end of period n calculated, until the absolute value of the difference between the calculated and set maximum drawdown starts to increase, which is a sign that in the previous iteration the difference was the smallest. In this way, the calculation could be performed for any number of periods, and much faster.

In the case when regenerations are planned, the number of regenerations m , and the value of the coefficient of reduction of local hydraulic losses due to regeneration y_{reg} are set, and within the periods between regenerations Equation (21) is solved for each of the given number of intermediate periods $p + 1$, for which y_{reg} equals 1, i.e. these are p quasi-regenerations that have no effect, but only represent the change in flow. LHR at the beginning of each intermediate period is calculated as LHR transferred from the previous intermediate period (Equation (20)).

$$LHR_n = LHR_{n-1} + KLHR_0 \frac{q_{n-1}}{q_0} \frac{t}{(m+1)(p+1)} \quad (20)$$

$$\Delta S = \left(LHR_n + KLHR_0 \frac{q_n}{q_0} \frac{t}{(m+1)(p+1)} + x \right) q_n \quad (21)$$

At the first intermediate period of each period between m real regenerations, LHR inherited from the preceding period between the regenerations is multiplied by y_{reg} (Equation (22)), and as the LHR acquired in the previous periods transferred to the period after the regeneration, used to calculate the LHR at the end of first intermediate period using Equation (20). If n is one, LHR is transferred from the last intermediate period (n equals $p + 1$) of the preceding period between the regenerations.

$$LHR_1 = \left(LHR_{p+1} + KLHR_0 \frac{q_{p+1}}{q_0} \frac{t}{(m+1)(p+1)} \right) y_{reg} \quad (22)$$

RESULTS

KLHR average values of flow per lateral, dissolved bivalent iron and redox potential

The results of the analysis of all periods between regenerations are given in Table 1 and Figure 2 (the graphs for all wells are available as supplementary material). The flows per lateral shown in the diagram in Figure 2 are the ones recorded when measuring levels, though there were usually more measurements in between, which were also taken into account when calculating average flow per lateral during a period without regeneration.

Results of regression analysis of KLHR

Dependence of KLHR on the concentration of dissolved bivalent iron and flow per lateral for 12 periods of operation of five wells with new, stainless steel laterals (Rb-5 m, Rb-8, Rb-15, Rb-16 and Rb-20) and the dominant biochemical colmatation mechanism is shown in Figure 3.

The best match to the data obtained on the basis of field measurements, when regression as a function of only the concentration of bivalent iron (in mg/dm³) is concerned, was obtained for the exponential function (Equation (23) and Figure 3).

$$KLHR = 0.1489 \cdot e^{0.4176 \cdot [Fe^{2+}]_{sr}}, \quad (23)$$

where the value of the coefficient of determination is 0.374. Because higher flows mean a greater influx of nutrients and ions from which mineral incrustations are formed, a higher flow under other conditions unchanged should correspond to a higher KLHR. Adding the dependence on the flow per lateral q (in L/s) produces a good match, significantly better than all other correlations, in the form of an exponential function of bivalent iron concentration multiplied by the power function of the flow per lateral, where the power of this function is approximately one (Equation (24) and Figure 3):

$$KLHR = q^{1.025} \cdot 0.00929 \cdot e^{0.7192 \cdot [Fe^{2+}]} \quad (24)$$

The value of the coefficient of determination is 0.874, which is a high and satisfactory value considering the inherent high level of uncertainty. The results obtained by applying the regression equation are shown by empty symbols, and the measured values are shown by full symbols in Figure 3. Equation (25) represents the dependence on the inlet velocity (in m/s) and the concentration of dissolved bivalent iron.

$$KLHR = v^{1.025} \cdot (1,000 \cdot l \cdot d \cdot \pi)^{1.025} \cdot 0.00929 \cdot e^{0.7192 \cdot [Fe^{2+}]} \quad (25)$$

Examination of the dependence of KLHR on the redox potential (standard hydrogen electrode) for 12 periods of operation of five wells with new, stainless steel laterals yielded the results of significantly worse quality and no sufficiently satisfactory correlation was achieved.

Dependence of KLHR on the concentration of dissolved bivalent iron and flow per lateral, for wells with biochemical colmatation as the dominant cause of aging of laterals, obtained by regression, is given by Equation (26). The value of the coefficient of determination is 0.717, which is also a high and satisfactory value. Dependence of KLHR on the concentration of dissolved bivalent iron and flow per lateral for 41 periods of operation of all 29 selected wells with, according to the assumption, dominant biochemical colmatation mechanism is illustrated in Figure 3.

$$KLHR = q^{0.8324} \cdot 0.01064 \cdot e^{0.7909 \cdot [Fe^{2+}]} \quad (26)$$

Because wells with older type of laterals have very large differences in permeability between laterals and along a lateral, it is difficult to make a connection between inlet velocities and flow per lateral, so no equation is presented that would correspond to Equation (25).

When the regression by bivalent iron concentration function only is concerned, the best match is also obtained for the exponential function, where the value of the coefficient of determination is 0.429 - (Equation (27)):

$$KLHR = 0.0489 \cdot e^{0.7922 \cdot [Fe^{2+}]} \quad (27)$$

Table 1 | Average values of flow per lateral, bivalent dissolved iron concentration, redox potential, *KLHR*, inlet velocities, critical flows and inlet velocities, and *KLHR* values obtained by the best fit regression, for periods of well operation between regenerations

	<i>KLHR</i> ₀ [m/ L/s/a]	$\Delta S/\Delta t$ [m/a]	<i>c</i> (Fe ²⁺) [mg/L]	<i>v</i> [m/s]	<i>Eh</i> [mv]	<i>q</i> [L/s]	<i>q</i> _{crit} ($\Delta S = 0.35$ m) [L/s]	<i>v</i> _{crit} ($\Delta S = 0.35$ m) [m/s]	<i>q</i> _{crit} ($\Delta S = 1.0$ m) [L/s]	<i>v</i> _{crit} ($\Delta S = 1.0$ m) [m/s]	<i>KLHR</i> _{reg} [m/(L/s/a)]
Rb-5 m	0.44	3.43	2.8	2.521·10 ⁻⁴	69.7	7.8	2.01	6.493·10 ⁻⁵	3.56	1.152·10 ⁻⁴	0.538
Rb-5 m	1.21	12.95	3.12	3.458·10 ⁻⁴	109.2	10.7	1.75	5.656·10 ⁻⁵	3.10	1.003·10 ⁻⁴	0.903
Rb-5 m	1.02	10.51	3.04	3.329·10 ⁻⁴	60.1	10.3	1.81	5.854·10 ⁻⁵	3.21	1.038·10 ⁻⁴	0.821
Rb-8	1.37	17.54	3.53	4.136·10 ⁻⁴	98.00	12.8	1.47	4.738·10 ⁻⁵	2.60	8.403·10 ⁻⁵	1.449
Rb-8	0.78	3.20	4.36	1.325·10 ⁻⁴	92.06	4.1	1.02	3.312·10 ⁻⁵	1.82	5.873·10 ⁻⁵	1.083
Rb-8	1.39	13.90	3.32	3.232·10 ⁻⁴	86.71	10	1.61	5.188·10 ⁻⁵	2.85	9.200·10 ⁻⁵	0.999
Rb-15	0.114	1.20	0.83	3.393·10 ⁻⁴	139.00	10.5	4.70	1.520·10 ⁻⁴	8.34	2.695·10 ⁻⁴	0.145
Rb-15	0.154	2.13	1.00	4.460·10 ⁻⁴	84.70	13.8	4.37	1.412·10 ⁻⁴	7.75	2.504·10 ⁻⁴	0.209
Rb-15	0.304	4.29	0.55	4.557·10 ⁻⁴	95.80	14.1	5.31	1.715·10 ⁻⁴	9.41	3.041·10 ⁻⁴	0.149
Rb-16	0.0471	0.98	0.47	6.703·10 ⁻⁴	128.25	20.7	5.49	1.775·10 ⁻⁴	9.74	3.148·10 ⁻⁴	0.193
Rb-20	0.318	3.28	1.64	3.329·10 ⁻⁴	77.30	10.3	3.32	1.071·10 ⁻⁴	5.88	1.900·10 ⁻⁴	0.271
Rb-20	0.492	9.00	1.25	5.914·10 ⁻⁴	71.90	18.3	3.92	1.268·10 ⁻⁴	6.96	2.248·10 ⁻⁴	0.322
Rb-2	0.12	0.91	1.41	2.456·10 ⁻⁴	164.00	7.6	3.66	1.183·10 ⁻⁴	6.49	2.098·10 ⁻⁴	0.176
Rb-3 m	0.033	0.35	1.51	3.425·10 ⁻⁴	116.00	10.6	3.51	1.133·10 ⁻⁴	6.22	2.010·10 ⁻⁴	0.251
Rb-4	0.05	0.97	0.13	6.269·10 ⁻⁴	275.00	19.4	6.36	2.056·10 ⁻⁴	11.28	3.646·10 ⁻⁴	0.139
Rb-4	0.021	0.37	0.34	5.623·10 ⁻⁴	162.00	17.4	5.81	1.878·10 ⁻⁴	10.30	3.330·10 ⁻⁴	0.150
Rb-6	0.334	1.60	0.53	1.551·10 ⁻⁴	134.90	4.8	5.35	1.730·10 ⁻⁴	9.49	3.068·10 ⁻⁴	0.060
Rb-6	0.422	3.46	0.56	2.650·10 ⁻⁴	115.70	8.2	5.28	1.708·10 ⁻⁴	9.37	3.028·10 ⁻⁴	0.095
Rb-8a	0.01	0.14	1.55	4.524·10 ⁻⁴	140.90	14.0	3.45	1.114·10 ⁻⁴	6.11	1.975·10 ⁻⁴	0.326
Rb-10	0.11	1.58	1.59	4.653·10 ⁻⁴	95.70	14.4	3.39	1.095·10 ⁻⁴	6.01	1.941·10 ⁻⁴	0.344
Rb-10	0.33	3.56	1.55	3.490·10 ⁻⁴	100.50	10.8	3.45	1.114·10 ⁻⁴	6.11	1.975·10 ⁻⁴	0.263
Rb-10 m	0.0825	0.93	0.76	3.652·10 ⁻⁴	120.90	11.3	4.85	1.566·10 ⁻⁴	8.60	2.778·10 ⁻⁴	0.146
Rb-21	0.093	0.97	0.09	3.361·10 ⁻⁴	245.30	10.4	6.47	2.092·10 ⁻⁴	11.48	3.709·10 ⁻⁴	0.080
Rb-23	0.0566	0.89	0.45	5.106·10 ⁻⁴	246.70	15.8	5.54	1.791·10 ⁻⁴	9.83	3.175·10 ⁻⁴	0.151
Rb-23	0.0173	0.19	0.31	3.555·10 ⁻⁴	176.10	11.0	5.89	1.902·10 ⁻⁴	10.44	3.373·10 ⁻⁴	0.100
Rb-26	0.058	0.75	1.45	4.169·10 ⁻⁴	159.20	12.9	3.60	1.163·10 ⁻⁴	6.38	2.062·10 ⁻⁴	0.282
Rb-27	0.325	2.37	2.29	2.359·10 ⁻⁴	141.70	7.3	2.50	8.092·10 ⁻⁵	4.44	1.435·10 ⁻⁴	0.340
Rb-27	0.255	2.68	1.74	3.393·10 ⁻⁴	72.70	10.5	3.18	1.026·10 ⁻⁴	5.63	1.820·10 ⁻⁴	0.298
Rb-41	0.138	1.08	2.84	2.521·10 ⁻⁴	90.10	7.8	1.97	6.382·10 ⁻⁵	3.50	1.132·10 ⁻⁴	0.556
Rb-42	0.702	3.30	2.39	1.519·10 ⁻⁴	121.90	4.7	2.40	7.750·10 ⁻⁵	4.25	1.374·10 ⁻⁴	0.255
Rb-47	0.256	1.05	1.64	1.325·10 ⁻⁴	92.80	4.1	3.32	1.071·10 ⁻⁴	5.88	1.900·10 ⁻⁴	0.126
Rb-48	0.127	0.50	1.27	1.260·10 ⁻⁴	91.70	3.9	3.89	1.257·10 ⁻⁴	6.90	2.229·10 ⁻⁴	0.090
Rb-49	0.045	0.39	1.85	2.779·10 ⁻⁴	119.40	8.6	3.03	9.785·10 ⁻⁵	5.37	1.735·10 ⁻⁴	0.276
Rb-52	0.182	1.33	2.39	2.359·10 ⁻⁴	101.10	7.3	2.40	7.750·10 ⁻⁵	4.25	1.374·10 ⁻⁴	0.369
Rb-52	0.361	2.56	1.53	2.294·10 ⁻⁴	53.90	7.1	3.48	1.123·10 ⁻⁴	6.17	1.992·10 ⁻⁴	0.182
Rb-59	0.039	0.25	0.68	2.036·10 ⁻⁴	143.1	6.3	5.02	1.621·10 ⁻⁴	8.90	2.875·10 ⁻⁴	0.084
Rb-64	0.069	0.19	1.05	8.725·10 ⁻⁵	89.80	2.7	4.28	1.382·10 ⁻⁴	7.58	2.451·10 ⁻⁴	0.056
Rb-65	0.31	1.32	2.15	1.380·10 ⁻⁴	91.70	4.3	2.66	8.596·10 ⁻⁵	4.72	1.525·10 ⁻⁴	0.195
Rb-79	0.158	0.55	1.74	1.131·10 ⁻⁴	109.50	3.5	3.18	1.026·10 ⁻⁴	5.63	1.820·10 ⁻⁴	0.120
Rb-83	0.0954	0.63	1.11	2.133·10 ⁻⁴	104.00	6.6	4.17	1.347·10 ⁻⁴	7.39	2.388·10 ⁻⁴	0.123
Rb-84	0.084	1.44	0.3	5.526·10 ⁻⁴	105.20	17.1	5.91	1.910·10 ⁻⁴	10.48	3.388·10 ⁻⁴	0.143
Rb-85	1.01	3.43	2.09	1.099·10 ⁻⁴	89.50	3.4	2.73	8.822·10 ⁻⁴	4.84	1.565·10 ⁻⁴	0.154

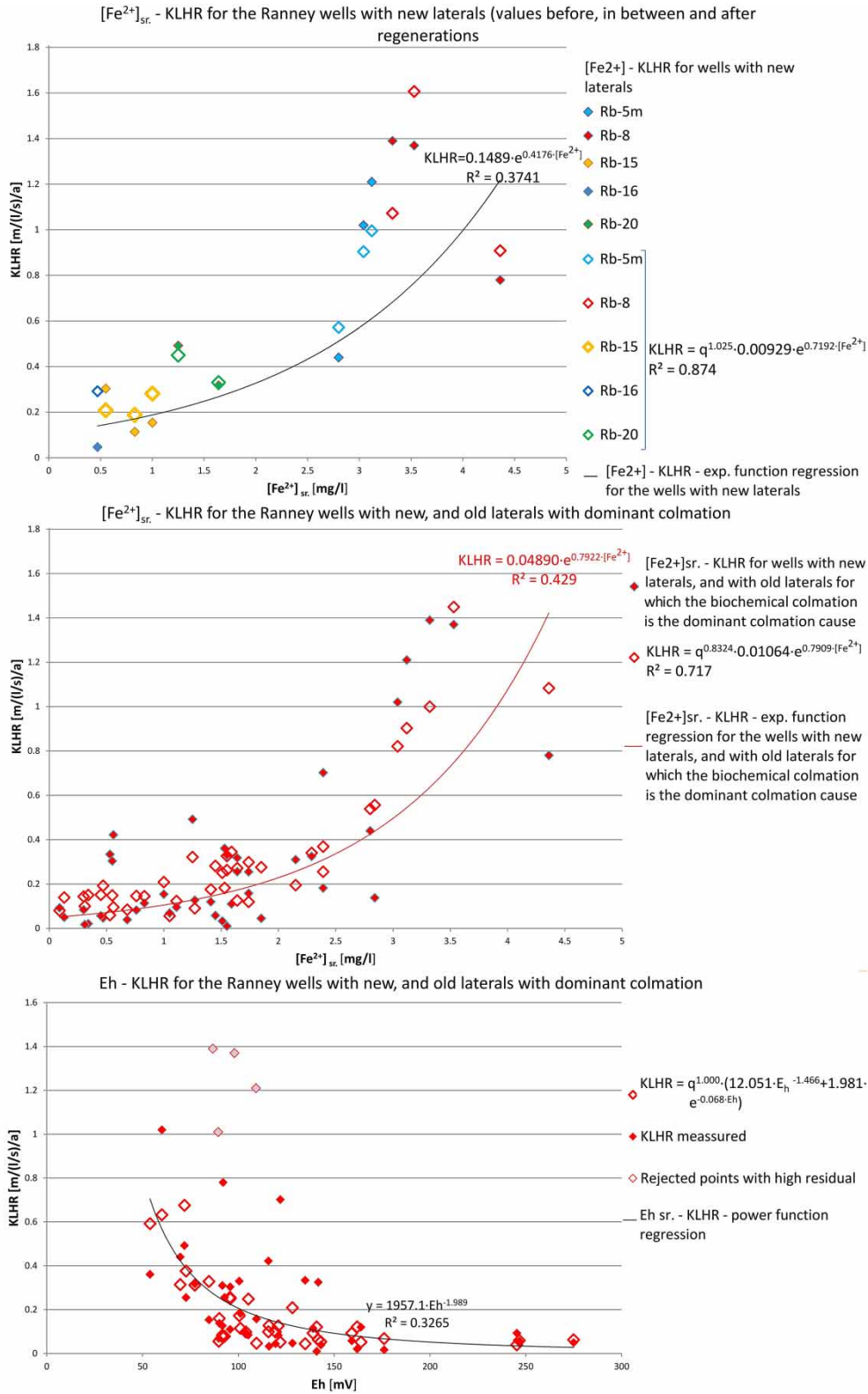


Figure 3 | Dependence of KLHR on the concentration of dissolved bivalent iron and flow per lateral for 12 periods of operation of five wells with new, stainless steel laterals (above), and for 41 periods in all 29 wells in which the dominant cause of well ageing is biochemical colmatation (middle), dependence of KLHR on the redox potential (standard hydrogen electrode) and flow per lateral for 37 periods in 26 wells with dominating biochemical mechanism of colmatations forming, with the matching regression functions and coefficients of determination.

The dependence of *KLHR* on the redox potential (standard hydrogen electrode) for 37 separate periods of operation of 26 wells with, presumably, dominating biochemical mechanism of colmation forming is shown in Figure 3. Four periods were rejected because the residuals were too large, which could not be corrected by any combination of exponential and degree functions. It was estimated that the best match to the data obtained on the basis of field measurements, at least in terms of the trend of sharp increase in *KLHR* with the drop in redox potential below 150 mV, was obtained for the function given by Equation (28).

$$KLHR = q^{1.000} \cdot (12.051 \cdot Eh^{-1.466} + 1.981 \cdot e^{-0.0680 \cdot Eh}) \quad (28)$$

The match with the measured values of redox potential was significantly less satisfactory than for dissolved bivalent iron concentration. The regression function with the highest value of coefficient of determination of 0.3265 is given by Equation (29):

$$KLHR = 1,957.1 \cdot Eh^{-1.989} \quad (29)$$

Regression analysis did not yield satisfactory results, and it can only be concluded with confidence from the distribution of *KLHR* values in relation to the redox potential that low *KLHR* values occur above 150 mV (matching well the observation in Dimkić *et al.* (2008) that the wells with values above 160 mV stand out with high yields, as well as the 150 mV limit adopted to label wells as being “good” or “bad” in Dimkić *et al.* (2011b), that between 85 and 150 mV *KLHR* values can range from 0.03 to 1.4 m/(L/s)/a, while below 85 mV all values are above 0.25 m/(L/s)/a.

Regeneration effects

During the 2006–2013 period, for which the necessary data are available (parameters significant for the effects of regenerations are given in Table 2), 27 regenerations were performed for which the value of y_{reg} could be determined, 13 of which were performed on wells belonging to the group with dominant biochemical colmation (italic font in Table 2), seven of which were the wells with new laterals (bold font in Table 2).

The effects of 27 regenerations (evaluated through the parameters significant for regeneration effects) performed during the period from 2006 to 2013, for which the necessary data are available, are presented in Table 2. Q_1 and Q_2 are well flows, q_1 and q_2 are flows per lateral, and q_{1ave} and q_{2ave} are the average flows per lateral during the periods before and after regeneration, respectively.

Of all the correlations examined, a weak linear correlation was found between flow per lateral before the regeneration and y_{reg} , and a slightly better power function correlation between local hydraulic losses before the regeneration and y_{reg} . The graphs are available as supplementary material.

DISCUSSION

Critical velocities and flows per lateral in relation to the concentration of dissolved bivalent iron

When planning the insertion of new laterals, one of the key decisions is the number of laterals. A larger number means a larger inlet area and thus lower inlet velocities, which also causes a decrease in *KLHR* according to Equation (24), but also means a higher cost (Dimkić & Pušić 2018). The higher the concentration of the dissolved bivalent iron, the higher the *KLHR* for the same flow per lateral, and it increases exponentially with the increase of the concentration. A practical way to take this into account is to define the maximum increase in local hydraulic losses on laterals during one year ($\Delta S_{max}a$), for example 0.35 or 1.0 m/a (Jaroslav Černi Institute 2010; Dimkić & Pušić 2014; Pušić & Dimkić 2017). For example, if the estimated exploitation time is 30 years, and the available loss of hydraulic potential is 10 m, then $\Delta S_{max}a$ is 0.33 m. Maximum flow per lateral (q_{max}) (Equation (31)), and maximum inlet velocity (v_{max}) (Equation (32)) are obtained from Equation (24), as the best correlation, by the derivation given in Equation (30). Dimensions of laterals – length l and diameter d – are expressed in meters.

$$\begin{aligned} \Delta S_{max}a &= KLHR \cdot 1 \cdot q_{max} + x \cdot q_{max} = q_{max} \cdot q_{max}^{1.025} \cdot 0.00929 \cdot e^{0.7192 \cdot [Fe^{2+}]} + x \cdot q_{max} = \\ &= q_{max}^{2.025} \cdot 0.00929 \cdot e^{0.7192 \cdot [Fe^{2+}]} + x \cdot q_{max} \end{aligned} \quad (30)$$

Table 2 | Parameters significant for the regeneration effect in 27 regenerations performed during the 2006–2013 period (from Mitrinović *et al.* (2018), with minor changes)

Well	Date	BEFORE REGENERATION									AFTER REGENERATION											
		Q ₁ [L/s]	q ₁ [L/s]	LHR ₁	KLHR ₁	q _{1ave} [L/s]	[Fe ²⁺] [mg/L]	Eh [mV]	No. of prev. reg.	No. of lat.	Q ₂ [L/s]	q ₂ [L/s]	LHR ₂	KLHR ₂	q _{2ave} [L/s]	br. dr.	Q ₂ /Q ₁	q ₂ /q ₁	q _{2sr} / q _{1sr}	KLHR ₂ / KLHR ₁	y _{reg} = LHR ₂ / LHR ₁	
1	Rb-15	28-12-09	41	13.7	0.5	0.114	10.5	0.83	139	0	3	76	25.3	0.36	0.154	13.9	3	1.85	1.85	1.32	1.35	0.72
2	Rb-15	20-04-12	25	8.3	0.71	0.154	13.8	1.0	84.7	1	3	49	16.3	0.39	0.304	13.5	3	1.96	1.96	0.98	1.97	0.57
3	Rb-19-1	30-04-09	61	8.7	0.48	0.0394	7.36	2.0	123.2	2	7	124	20.7	0.16	0.1325	8	6	2.03	2.37	1.09	3.36	0.33
4	Rb-23	29-09-08	125	15.6	0.195	0.0566	15.8	0.45	246.7	4	8	88	11.0	0.13	0.0173	11.7	8	0.71	0.70	0.74	0.31	0.67
5	Rb-29	16-06-10	15	1.9	16	2.87	1.91	1.61	95.5	2	8	57	14.3	0.3	0.856	4.8	4	3.8	7.60	2.51	0.30	0.019
6	Rb-2 m	29-12-11	19	4.8	0.71	0.014	10.6	0.97	170.3	5	4	72	24.0	0.32	0.078	21.6	3	3.79	5.05	2.04	5.57	0.35
7	Rb-3	01-07-10	13	4.3	6.75	1.87	5.33	1.41	125	5	3	10.5	5.3	1.1	1.43	4.2	2	0.81	1.21	0.79	0.76	0.16
8	Rb-4	19-04-08	164	20.5	0.188	0.05	19.4	0.13	275	2	8	206	25.8	0.062	0.021	18.9	8	1.26	1.26	0.97	0.42	0.33
9	Rb-5 m	20-08-10	24	6.0	1.2	0.443	7.8	2.8	69.7	0	4	95	23.8	0.2	1.21	8.5	4	3.96	3.96	1.09	2.73	0.21
10	Rb-5 m	27-06-12	17	4.3	0.7	1.21	10.7	3.12	109.2	1	4	52	13.0	2.5	1.02	8	4	3.06	3.06	0.75	0.84	0.28
11	Rb-6	01-03-11	24	2.4	2.2	0.334	4.8	0.53	134.9	4	10	33	5.5	0.45	0.422	7.7	6	1.38	2.29	1.60	1.26	0.2
12	Rb-52	16-04-10	17	2.1	1.25	0.182	7.3	2.39	101.1	2	8	56	8.0	0.66	0.361	6.1	7	3.29	3.76	0.84	1.98	0.54
13	Rb-53	30-04-10	10.5	2.1	1.4	0.292	9.26	3.1	113.2	3	5	16.8	3.4	0.7	0.665	4.6	5	1.6	1.60	0.50	2.28	0.5
14	Rb-79	23-09-08	28	3.5	5.8	0.01	3.5	1.74	109.5	2	8	11	1.4	1	0.158	3.4	8	0.39	0.39	0.97	15.80	0.17
15	Rb-8	22-05-08	16	5.3	1.3	1.37	12.8	3.53	98	1	3	18	6.0	1.3	0.781	4.5	3	1.125	1.13	0.35	0.57	1
16	Rb-8	23-03-12	5	1.7	4.25	0.781	4.1	4.36	92.06	2	3	69	23.0	0.4	1.39	7.5	3	13.8	13.80	1.83	1.78	0.094
17	Rb-9	01-07-10	9	4.5	4.4	0.738	5.5	2.8	83	2	2	6.3	6.3	1.25	0.508	5.9	1	0.7	1.40	1.07	0.69	0.28
18	Rb-10	26-02-10	31	5.2	0.46	0.11	14.4	1.59	95.7	7	6	59	11.8	0.09	0.33	10.2	5	1.9	2.28	0.71	3.00	0.2
19	Rb-11-1	01-07-08	50	6.3	1.4	0.022	6.38	2.82	85	2	8	45	11.3	0.7	0.258	7.2	4	0.9	1.80	1.13	11.73	0.5
20	Rb-12-3	16-04-08	40	8.0	1.13	0.01	5.92	2.1	110	3	5	44	22.0	0.27	0.181	12	2	1.1	2.75	2.03	18.10	0.24
21	Rb-81	20-11-09	42	7.0	4.3	0.01	4.69	1.01	105.3	1	6	60	15.0	0.15	0.556	6.1	4	1.43	2.14	1.30	55.60	0.035
22	Rb-90	12-11-10	28	4.0	4.4	0.652	5.8	2.27	113	1	7	49	9.8	1.9	0.679	5.4	5	1.75	2.45	0.93	1.04	0.43
23	Rb-13	06-01-11	55	7.9	0.085	0.0122	7	-	-	5	7	112	16.0	0.06	0.0346	8.1	7	2.04	2.04	1.16	2.84	0.71
24	Rb-13-1	01-07-11	13.5	3.4	4.7	0.694	4.77	1.09	92	3	4	12.5	6.3	1.8	0.638	4.7	2	0.93	1.85	0.99	0.92	0.375
26	Rb-20	05-06-12	12	3.0	1.3	0.318	10.3	1.64	77.3	0	4	85	21.3	0.29	0.492	16.8	4	7.08	7.08	1.63	1.55	0.22
27	Rb-40	05-06-08	20.5	5.1	2.4	0.648	7.61	0.94	168.6	3	4	21	5.3	2.7	0.128	4.2	4	1.02	1.02	0.55	0.20	1.13
avg																	2.45	2.95	1.15	5.27	0.40	

Equation (30) can be solved, or, if the resistance of the aquifer is neglected as significantly smaller than the final local hydraulic resistance, q_{max} can be calculated as:

$$q_{max} = q_{crit} = \left(\frac{\Delta S_{max} a}{0.00929 \cdot e^{0.7192 \cdot [Fe^{2+}]}} \right)^{1/2.025} \quad (31)$$

while v_{max} (in m/s) can be calculated as:

$$v_{max} = v_{crit} = \frac{\left(\frac{\Delta S_{max} a}{0.00929 \cdot e^{0.7192 \cdot [Fe^{2+}]}} \right)^{1/2.025}}{1,000 \cdot l \cdot d \cdot \pi} \quad (32)$$

Inlet velocities and flows per lateral with matching concentrations of dissolved bivalent iron for wells with new laterals (above) and laterals with dominant biochemical colmation (below) are shown in Figure 4. The dependence of critical velocities and critical flows per lateral on dissolved bivalent iron concentration according to Equations (31) and (32), with critical values from other papers, is also shown. In addition, velocities were singled out in those wells where the multiplication of the mean flow per lateral with KLHR resulted in an annual decrease in level of approximately 1.0 m. This is generally not an actual increase (as the level is usually kept close to minimum and the flow is slowly decreasing).

Regarding wells with dominant biochemical colmation, in Figure 4 (below), it can be observed that the zones of recommended velocities/flows per lateral according to Jaroslav Černi Institute (2010) match quite well the velocities/flows per lateral of 12 periods with an annual decrease in level of about 1.0 m (Table 1), and with the exponential function obtained by regression. The values of velocities and flows per lateral calculated by the exponential function obtained by regression for these 12 periods are somewhat higher than the critical values (for an annual decrease in level of 1.0 m) obtained on the basis of Equation (26). Recommended velocities/flows according to Dimkić & Pušić (2014) for $\Delta S_{max} a = 0.35$ m/a are in increasing percentage higher than the values obtained on the basis of Equation (26) below the dissolved iron concentration of 2.2 mg/L and in decreasing percentage lower above 2.2 mg/L. The differences are not surprising given that the correlation for KLHR and redox potential in that paper was derived based on the data from three other groundwater sources and one drainage system in the Danube, Tisza, and Velika Morava alluviums, and that the correlation with bivalent iron concentration was performed using a correlation between bivalent iron concentration and redox potential (Dimkić & Pušić 2014).

In wells with the new laterals, the critical velocities and flows per lateral obtained from Equations (31) and (32) for an annual level drop of about 1.0 m match the lower part of the zones of recommended velocities/flows per lateral according to Jaroslav Černi Institute (2010). The velocities and flows per lateral calculated using the exponential function obtained by regression (for only two periods in wells with new laterals having an annual level drop of about 1.0 m) are in increasing percentage higher than the critical velocities/flows obtained from Equations (31) and (32) above the dissolved iron concentration of 1.0 mg/L and in decreasing percentage lower below 1.0 mg/L (Figure 4).

Critical velocities and flows per lateral in relation to the redox potential

Starting from Equation (28), mathematical expressions for the corresponding maximum flow per lateral (q_{max}) (Equation (34)), and maximum inlet velocity (v_{max}) (Equation (35)) are obtained using derivation given in Equation (33).

$$\Delta S_{max} a = KLHR \cdot l \cdot q_{max} = q_{max}^{2.000} \cdot (12.05 \cdot Eh^{-1.466} + 1.981 \cdot e^{-0.0680 \cdot Eh}) \quad (33)$$

$$q_{max} = \left(\frac{\Delta S_{max} a}{12.051 \cdot Eh^{-1.466} + 1.981 \cdot e^{-0.0680 \cdot Eh}} \right)^{1/2.000} \quad (34)$$

$$v_{max} = \frac{\left(\frac{\Delta S_{max} a}{12.051 \cdot Eh^{-1.466} + 1.981 \cdot e^{-0.0680 \cdot Eh}} \right)^{1/2.000}}{1,000 \cdot l \cdot d \cdot \pi} \quad (35)$$

Figure 5 shows a fairly good match between the values of calculated critical velocities and flows per lateral for a maximum annual level drop of 1 m with velocities and flows per lateral for 12 periods with an annual increase in level drop of about

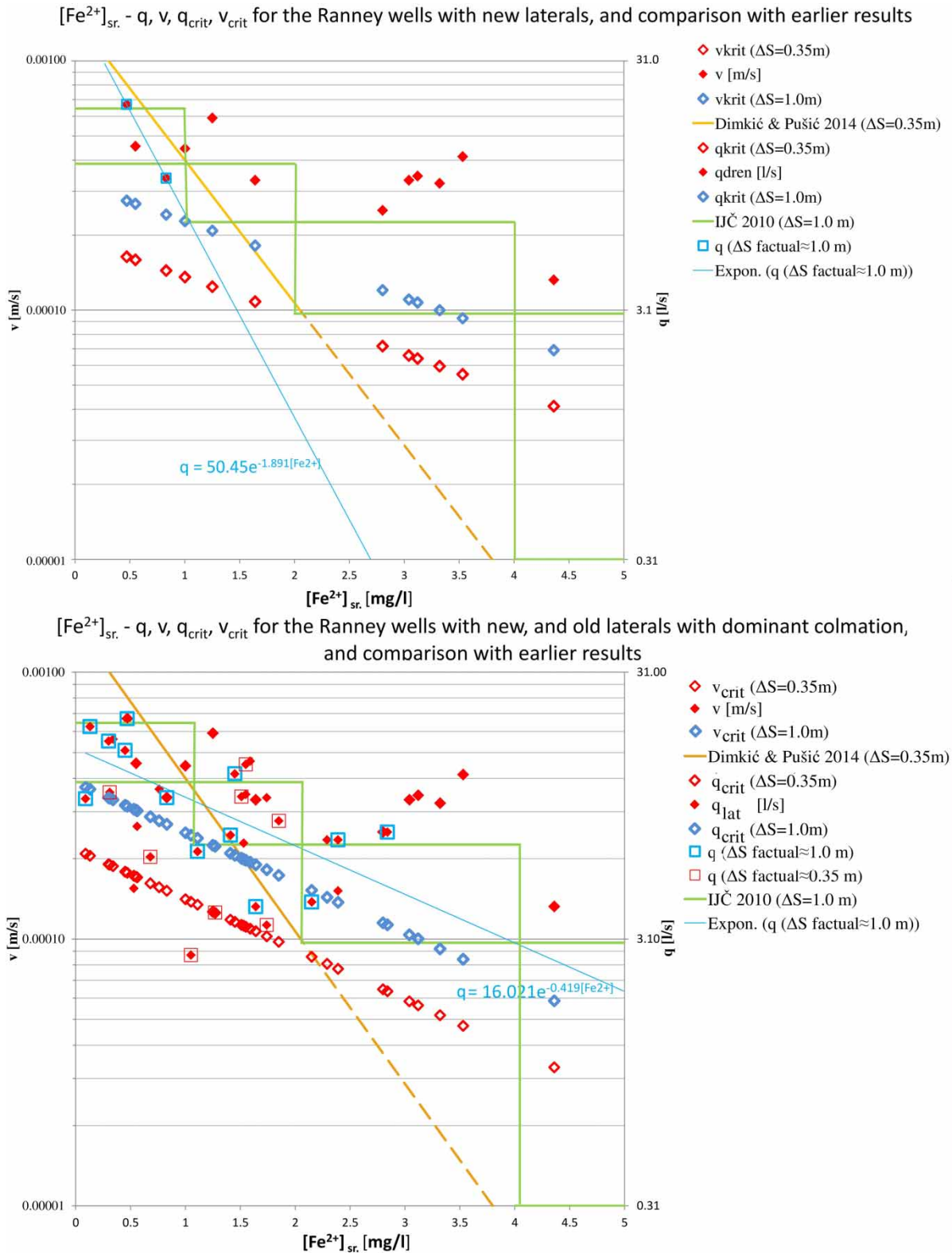


Figure 4 | Dependence of velocities and flows per lateral, as well as critical velocities and critical velocities per lateral (for annual decrease of levels of 0.35 and 1.0 m) on concentration of dissolved ferrous iron (Equations (31) and (32)), with critical values from other research also shown, for the new laterals only (above), and laterals with dominant biochemical colmatation (below). Green rectangles are the zones of recommended velocities/flows for $\Delta S_{maxa} = 1.0$ m/a according to Jaroslav Černi Institute (2010).

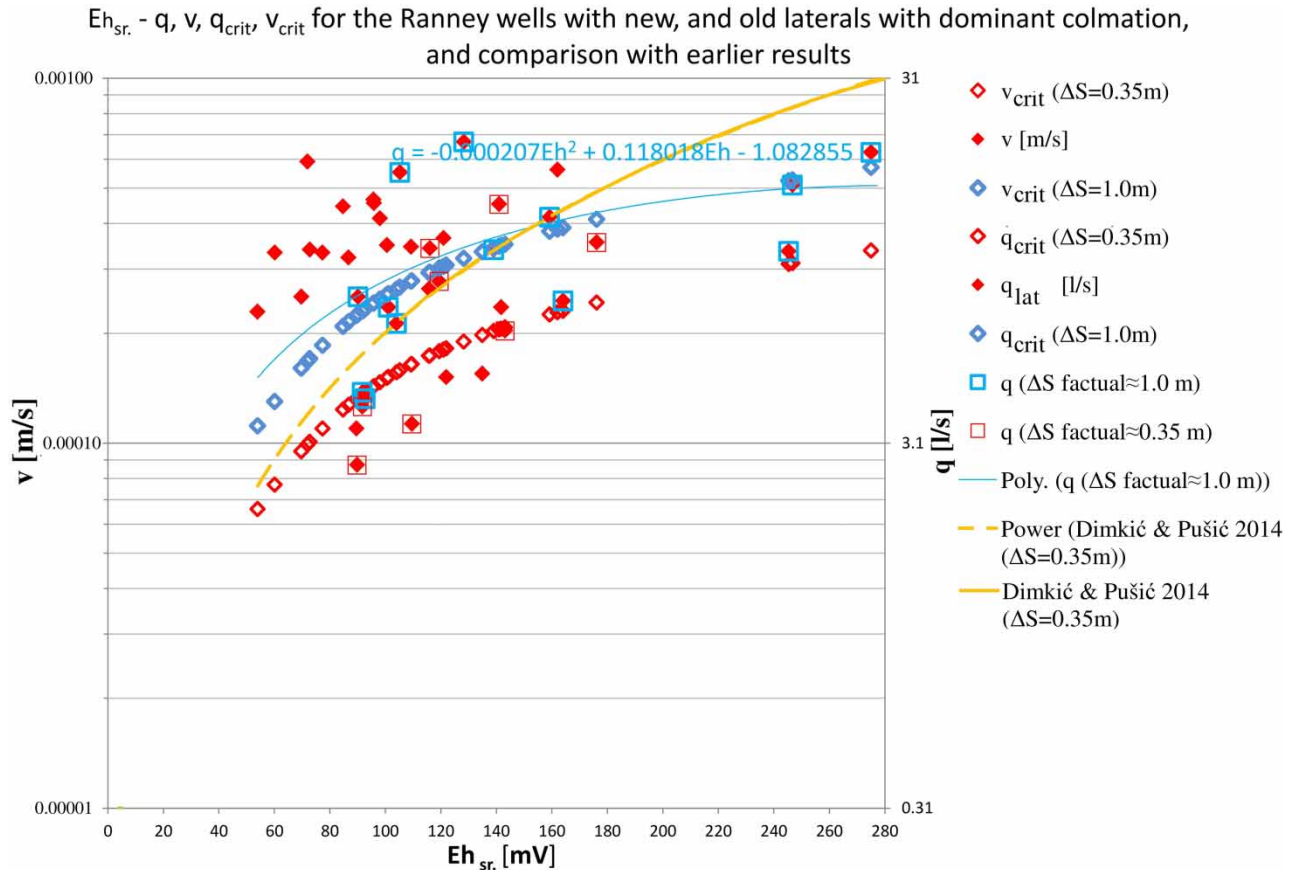


Figure 5 | Critical velocities and flows per lateral (for annual level drops of 0.35 and 1.0 m) in relation to the redox potential (Equations (34) and (35)), for new, and laterals with dominant biochemical colmation, showing critical values from other research.

1.0 m (Table 1). The match with the regression-obtained polynomial for these 12 wells (blue curve in Figure 5) is very good. Recommended velocities/flows per lateral according to Dimkić & Pušić (2014) for $\Delta S_{max} = 0.35$ m/a (orange curve in Figure 5) are in increasing percentage higher than the values obtained on the basis of Equations (34) and (35) above 40 mV, which is not surprising considering that the correlation in that paper was performed on the basis of averaging the values over five ranges of redox potential, for a different set of wells at the Belgrade groundwater source and using additional vertical wells from the groundwater sources along Velika Morava river.

Upon comparison with the critical velocities and flows per lateral calculated on the basis of the concentration of bivalent iron, it can be noticed that the values are higher when calculated using redox potential. Since the quality of the regression is much better for correlations with bivalent iron concentration, it is advisable to use them, except for wells with high redox potentials, above 150 mV, as the *KLHRs* calculated using Equation (28) match data based on field measurements better in this range, than the ones calculated using Equations (24) and (26) at low concentrations of ferrous ions corresponding to high values of redox potential (Figure 3).

The decision on the maximum acceptable annual level drop depends also on the assessment of whether and how often regenerations will be performed, as well as the prognosis of their effects.

Analysis of well operation using software model

Constant flow in between the regenerations

The simulation procedure was performed for the cases of 1, 4 and 7 regenerations for all combinations of $KLHR_0$, aquifer resistance x (drawdown per unit flow per lateral) and maximum available drawdown ΔS_{max} values, for each y_{reg} value. Using a macro written in the Visual Basic for Applications programming language, the calculations in Excel Solver are started automatically for all 4,851 combinations of number of regenerations (3 values), y_{reg} (11 values), $KLHR_0$ (7 values), x (7 values)

and ΔS_{max} (3 values) and the results are recorded in separate spreadsheets (for cases with 1, 4 and 7 regenerations) and in one spreadsheet with complete results. The ratio of total volumes of pumped water during the observed period with (Q_1) and without regenerations (Q_0) was also calculated from the flows per lateral recorded for each combination of key parameters.

In the next step, a regression analysis of the obtained results was performed to obtain the function correlating Q_1/Q_0 with the aquifer resistance x (drawdown per unit flow per lateral), the coefficient of reduction of local hydraulic losses due to regeneration (y_{reg}) and the number of regenerations (n). The best correlation was achieved for the function given by Equation (36):

$$\frac{Q_1}{Q_0} = (0.19603 \cdot n - 0.5382) \cdot (0.20378 \cdot x - 1.406575 - 0.94751 \cdot (5 - n) \log(y_{reg} + 1) - 1.23763) \quad (36)$$

For one regeneration during the observed period total volume of pumped water during that period is 11% ($y_{reg} = 1.0$) to 41% ($y_{reg} = 0.0$) larger than the total flow without regeneration (using Equation (33) – 0% ($y_{reg} = 1.0$) to 59% ($y_{reg} = 0.0$)), for four regenerations 21% ($y_{reg} = 1.0$) to 123% ($y_{reg} = 0.0$) larger (using Equation (33) – from 21% ($y_{reg} = 1.0$) to 120% ($y_{reg} = 0.0$)), for seven regenerations 24% ($y_{reg} = 1.0$) to 182% ($y_{reg} = 0.0$) larger (using Equation (36) – 44% ($y_{reg} = 1.0$) to 164% ($y_{reg} = 0.0$)).

Critical velocities and flows per lateral. In case of regenerations being performed, reductions in *LHR* due to regenerations allow for higher flows and inlet velocities than maximum ones according to Equations (31), (32), (34) and (35). The length of the period before the first regeneration (and between other regenerations) was taken as equal to the length of the entire analyzed period t divided by the number of regenerations plus one. In Equation (37), the expressions for maximum drawdown for the entire period of length t , as well as for the period before the first regeneration are given, taking into account that *KLHR* changes proportionally to the change in lateral flow:

$$\Delta S_{max} = (x + KLHR_0 t) \cdot q_0 = \left(x + KLHR_0 \frac{q_1}{q_0} \frac{t}{n + 1} \right) \cdot \frac{q_1}{q_0} q_0 \quad (37)$$

If the resistance of the medium x is small in relation to the local resistances, it can be neglected, which leads to expression (38):

$$q_0 \sqrt{n + 1} = q_1 \quad (38)$$

Based on Equation (35), the maximum allowable lateral flows and input velocities according to Equations (31), (32), (34) and (35) can be multiplied by approximately the square root of the number of regenerations increased by 1, to obtain the maximum allowable values in the first period of well operation. Based on the results of the calculation performed using a macro written in the Visual Basic for Applications and Excel Solver, in the already explained way, it was found that when the mean value of the coefficient of reduction of local hydraulic losses due to regeneration y_{reg} is about 0.5 ± 0.1 , the lateral flow after all subsequent regenerations should be about 75% of that before the first regeneration.

Constant level in between the regenerations

Figure 6 (top) shows how the level inside the well changes at a constant flow between the regenerations. In practice, the operation of the well pump is usually regulated in such way as to keep the level constant, hence the flow changes over time (Figure 6 (bottom)).

Figure 6 (bottom) shows the change in flow per lateral during a 30-year period, for the case when there are no regenerations and the maximum drawdown is reached after 30 years (red line) of constant flow, and for the cases when pumping at maximum drawdown is simulated by dividing the 30 year period to 10, 25, 50, and 100 intermediate periods (violet, blue, green, and yellow curves, respectively). It can be observed that the initial, high flows can be obtained by calculation only if the intermediate periods are short enough (Figure 6). Approximately 30% higher maximum flows can be obtained in approximately twice as long periods. The shorter the intermediate period – the higher the flow and the calculated drawdown due to the aquifer losses and accumulated *LHR*, the lower the remaining available drawdown for the increase of local losses. At very short lengths of intermediate periods, level in the well practically does not change, and gradual decrease in flow at constant level is replicated in the calculation. The ratio between the total extracted water when pumping with a constant, maximum

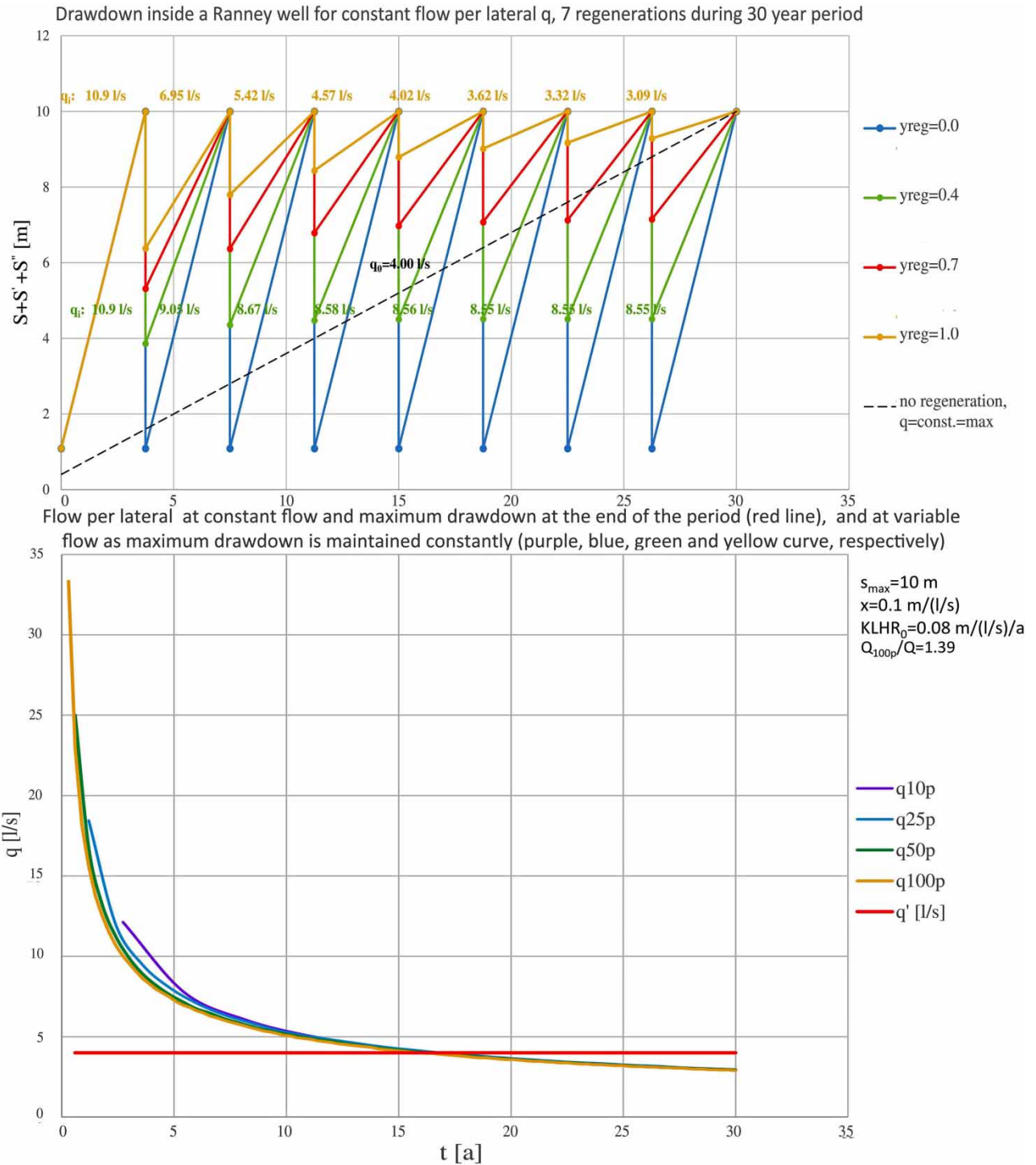


Figure 6 | Drawdown in the well shaft over a period of 30 years, for the case of 7 regenerations (broken black line for no regeneration), for the constant flow between the regenerations, $x = 0.1$ m/(L/s), $KLHR_0 = 0.08$ m/(L/s)/a (above). Flow per lateral during the 30 year period of well operation without regeneration at constant flow and maximum drawdown at the end of the period (red line), results of simulation of pumping period during which the maximum drawdown is maintained, by dividing it into 10, 25, 50 and 100 shorter periods – purple, blue, green and yellow curve, respectively, $x = 0.1$ m/(L/s), $KLHR_0 = 0.08$ m/(L/s)/a (below).

drawdown (simulated over 100 intermediate periods) and pumping at constant flow in such way that the maximum drawdown is reached after 30 years, Q_{100p}/Q , is equal to 1.39 for the particular combination of the resistance of aquifer and kinetics of local hydraulic losses (Figure 6).

The change in the flow per lateral during the 30 year well operation period is shown in Figures 7 and 8, in case there is no regeneration and the maximum drawdown is achieved after 30 years (red line), in case regenerations are performed (4 or 7 during the period in consideration) with maximum drawdown (10 m) achieved at the end of the period between regenerations (yellow line), and in the case of regenerations being performed and drawdown always equal to the maximum value (gray curve), all three cases simulated for different values of $KLHR_0$ and aquifer resistance x . The ratio between the total extracted water with drawdown constantly kept equal to the maximum between regenerations (simulated over 400 intermediate periods in total), and with maximum drawdown reached just before the next regeneration, marked as Q_{400p}/Q , ranges from 1.12 to 1.25 in the case of 4 regenerations, while in the case of 7 regenerations ranges from 1.10 to 1.18.

Pumping constantly at maximum drawdown according to the model produces some 20% larger volume of total extracted water.

CONCLUSION

By isolating wells in which colmation of laterals is a process that predominantly causes local hydraulic resistance increase, as well as by treating periods separated by regenerations as independent, satisfactory correlations between $KLHR$ and dissolved bivalent iron concentration and flow per lateral were obtained (and to a certain extent useful correlation between $KLHR$ and redox potential and flow per lateral).

Critical velocities and flows per lateral, given by Equations (31) and (32) in general match the values from previous papers and studies, mostly IJC (2010), while critical velocities/flows according to Dimkić & Pušić (2014) for $\Delta S_{max} = 0.35$ m/a for a dissolved iron concentration of 2.2 mg/L are the same as obtained by Equations (31) and (32), but they differ more and more

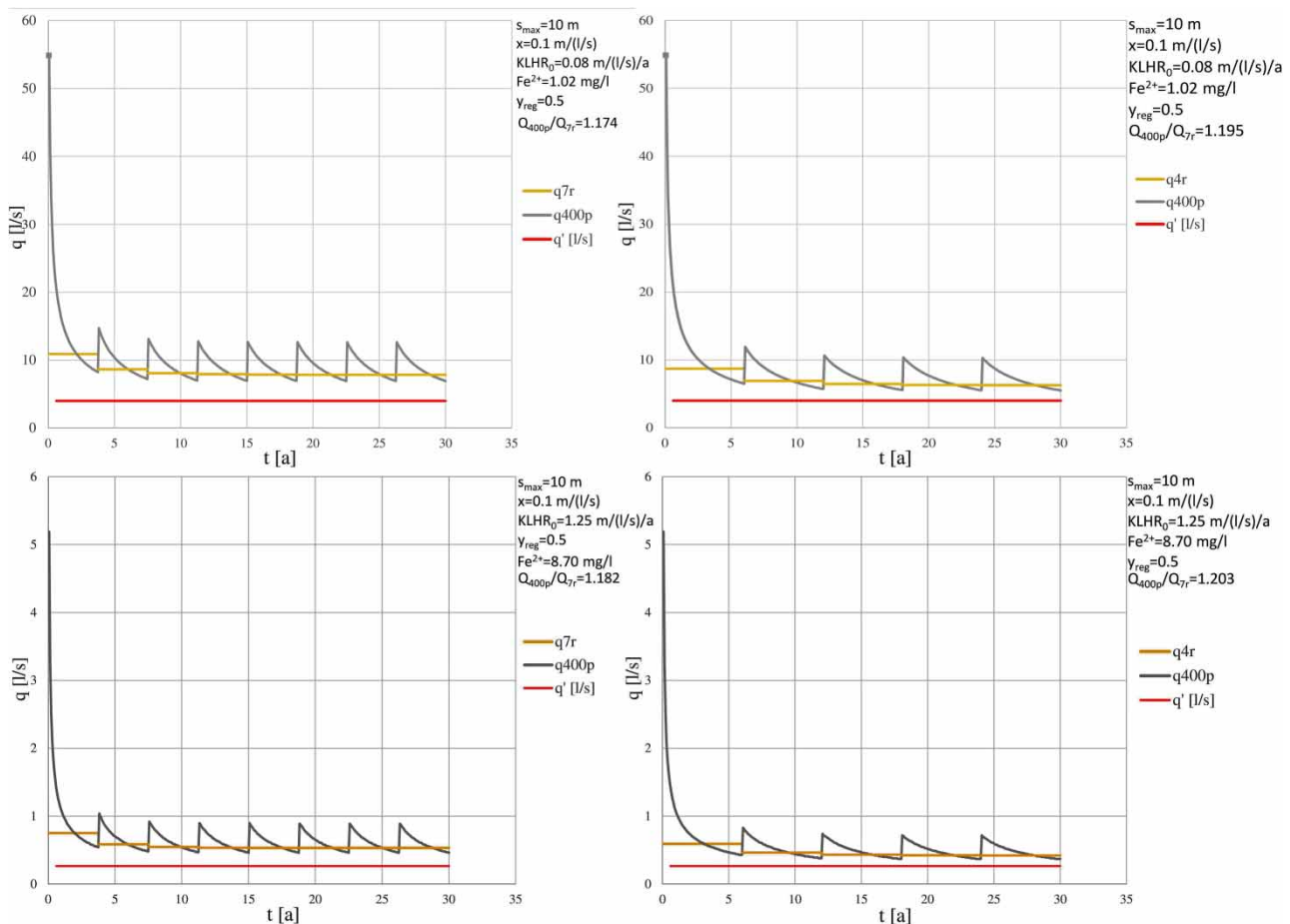


Figure 7 | Change in flow per lateral, in case there is no regeneration and maximum drawdown is reached after 30 years (red line), in case regenerations are performed and maximum drawdown is reached at the end of the period between regenerations (yellow line), and in case regenerations are performed and the drawdown is kept equal to the maximum (gray curve), for $x = 0.1$ m/(L/s), and $KLHR_0 = 0.08$ m/(L/s)/a (above) and 1.25 m/(L/s)/a (below) for 4 regenerations (right) and 7 regenerations (left).

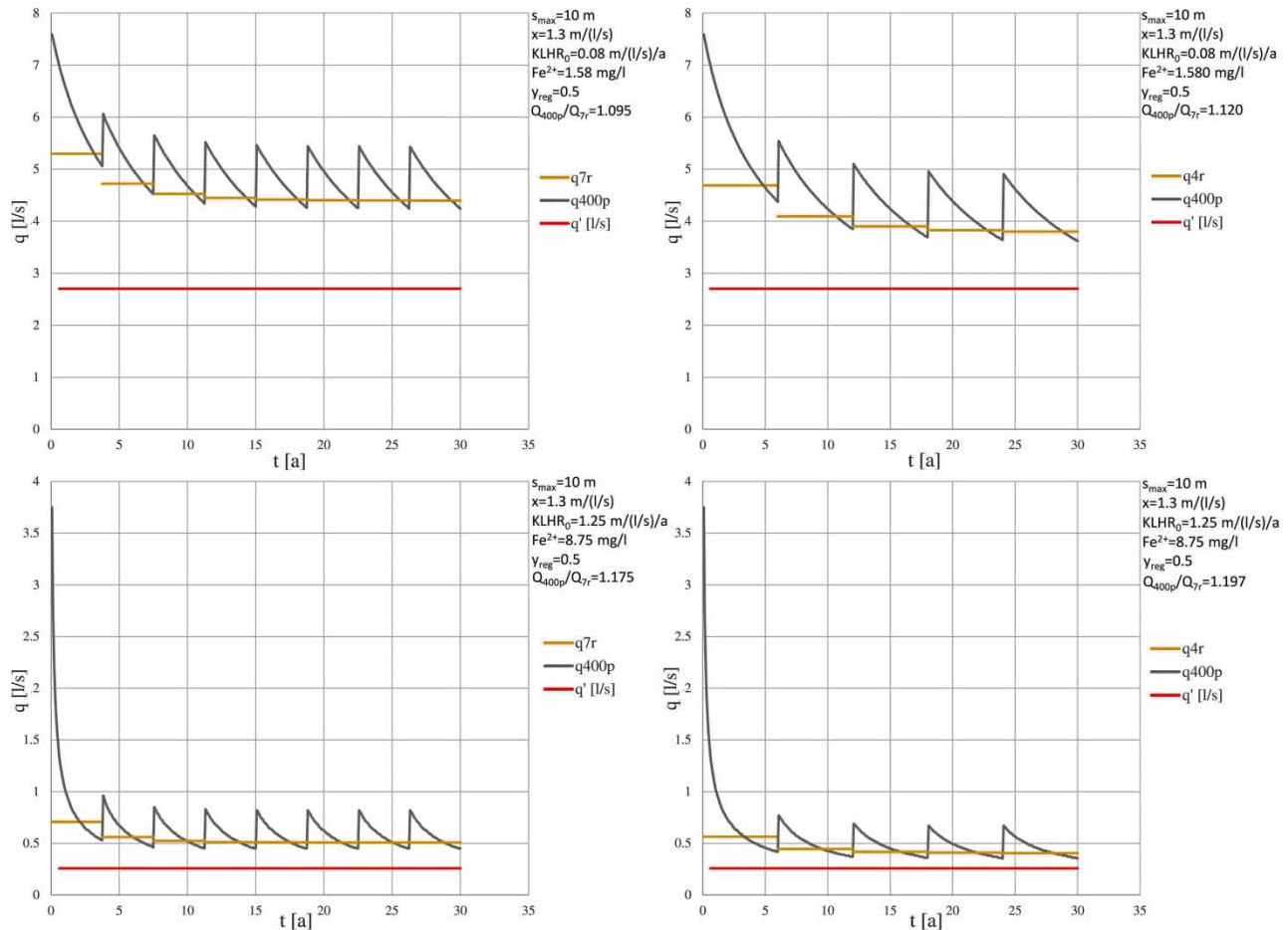


Figure 8 | Change in flow per lateral, in case there is no regeneration and maximum drawdown is reached after 30 years (red line), in case regenerations are performed and maximum drawdown is reached at the end of the period between regenerations (yellow line), and in case regenerations are performed and the drawdown is kept equal to the maximum (gray curve), for $x = 1.3 \text{ m/(L/s)}$, and $KLHR_0 = 0.08 \text{ m/(L/s)/a}$ (above) and 1.25 m/(L/s)/a (below) for 4 regenerations (right) and 7 regenerations (left).

as the concentrations get higher or lower than this value, which, having in mind that the average concentration for the analyzed periods is 1.55 mg/L , is not too large a deviation.

The impact of regenerations on the total amount of pumped water is large. In the case of maximum analyzed number of 7 regenerations (approximately once in 4 years for a period of 30 years) at $y_{reg} = 0.4$, which is the average value for analyzed regenerations (Table 2) at Belgrade groundwater source, it is two times larger than in the case of no regeneration. The correlation connecting the ratio of the amount of pumped water with (Q_I) and without regeneration (Q_0) and the resistance of the aquifer x , the coefficient of reduction of local hydraulic losses due to regeneration y_{reg} and the number of regenerations n is given by Equation (36). According to Equation (38), the maximum recommended values of velocities and flows per lateral (obtained from Equations (31), (32), (34) and (35)) at the beginning of well operation can be multiplied by approximately the square root of the number of regenerations increased by 1. After regeneration, the flow per lateral should be about 75% of that before the first regeneration if y_{reg} is about 0.5 ± 0.1 . The effects of regeneration cannot be predicted, only weak correlations have been established between flow per lateral before regeneration and y_{reg} , as well as between local hydraulic losses before regeneration and y_{reg} , meaning that the effects of regeneration increase moderately with the magnitude of local resistances. The approach that can be proposed is to use the y_{reg} value for design calculations by 0.1–0.2 higher than the average for the groundwater source.

In practice, the well pump is operated in such a way that the level is kept constant and the flow decreases over time. This mode of operation is simulated by dividing the period between regenerations into a large number of smaller periods in which the flow is constant, and it changes between these periods in such a way that the level at the end of the period is the same as at the end of the previous one. In this way, the phenomenon of high initial flows, which decline very quickly, is successfully replicated

(Figures 6–8). The ratio between the volume of total extracted water for the case when the drawdown is constantly kept equal to maximum (simulated over 400 intermediate periods) and when the flow between the regenerations is kept constant (as the level in the well decreases), is between 1.12 and 1.25 for 4 regenerations, while for the case of 7 regenerations is between 1.10 and 1.18.

Providing that the effects of regeneration are the same, after several regenerations the calculations show that the flow does not decrease further as LHR has become large enough that the percentage eliminated by regeneration becomes equal to the resistance accumulated during the period after the previous regeneration.

The developed software model can be used to simulate different scenarios for number of laterals, well operation mode and regeneration, or Equations (31), (36) and (38) can be used, together with ratios that pertain to the differences between the constant level and constant flow mode, to predict the well yields and total extracted volumes.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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