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8th International Congress of the Serbian Society of Mechanics

June 28-30, 2021
Kragujevac, Serbia

Hellenic Society for Theoretical & Applied Mechanics (HSTAM)

СДРМ
СРПСКО ДРУШТВО ЗА РАЧУНСКУ МЕХАНИКУ
SERBIAN SOCIETY FOR COMPUTATIONAL MECHANICS

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGICAL DEVELOPMENT

The 8th International Congress of the Serbian Society of Mechanics Kragujevac, Serbia, June 28-30, 2021

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Welcome Message

Dear colleagues,

It is a great pleasure for us to welcome you all at *the* ***8th International Congress of the Serbian Society of Mechanics*** in Kragujevac, Serbia Well-known for its culture, history and industrial heritage, Kragujevac was the first capital of modern Serbia and the place where the first constitution in the Balkans was proclaimed. Today, we are more than proud to say that Kragujevac is also becoming one of the scientific capitals in the region.

In this very difficult time of the COVID-19 pandemic, we decided to make this congress a hybrid event combining physical and online sessions, so that everyone interested can join us despite the obstacles we have all been facing for more than a year now.

8th International Congress of the Serbian Society of Mechanics aims to bring together leading academic scientists, researchers and research scholars to exchange and share experiences and research results on various aspects of *Theoretical and Applied Mechanics*. It will bring an interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, theories, algorithms, as well as practical challenges encountered and solutions adopted in the fields of *Classical Mechanics, Solid and Fluid Mechanics, Computational Mechanics, Biomechanics, Applied Mathematics and Physics, Structural Mechanics and Engineering.*

The Congress is organized by the Serbian Society of Mechanics (SSM) in partnership with: Faculty of Engineering, University of Kragujevac, Faculty of Mechanical Engineering, University of Belgrade, Faculty of Technical Science, University of Novi Sad, Faculty of Mechanical Engineering, University of Niš, Hellenic Society of Theoretical and Applied Mechanics, Institute of Information Technology Kragujevac, University of Kragujevac, with the support of the Serbian Ministry of Education, Science and Technological Development, Serbian Academy of Sciences and Arts and Serbian Society for Computational Mechanics.

Six distinguished plenary speakers will deliver lectures:

1. Prof. Georgios E. Stavroulakis – Technical University of Crete, Greece
2. Prof. Themis Exarchos – Ionian University, Corfu, Greece
3. Prof. Mihailo R. Jovanović – University of Southern California, USA
4. Prof. Ricardo Ruiz Baier – Monash University, School of Mathematics, Clayton, Australia
5. Dr Božidar Jovanović – MISANU, Serbia
6. Dr Marko Janev – MISANU, Serbia

The Congress encompasses six main topics: General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies, Biomechanics, Control and Robotics, Interdisciplinary and Multidisciplinary Problems.

Also, there are four Mini-Symposia:

- M1: 5th Serbian-Greek Symposium on Advanced Mechanics
  Chairs: Prof. Georgios Stavroulakis, President of HSTAM, Greece; Prof. Nenad Filipović, President of SSM, Serbia
- M2: Turbulence
  Chair: Prof. Đorđe Čantrak, University of Belgrade, Serbia
- M3: Mathematical Biology and Biomechanics
  Chair: Dr. Anđelka Hedrih, MI SANU, Serbia
- M4: Nonlinear Dynamics
  Prof. Julijana Simonović, University of Niš, Serbia

Within the Congress, we are also very proud to organize the 5th Serbian-Greek Symposium on *Current and Future Trends in Mechanics*. The Symposium is organized by the Serbian Society of Mechanics (SSM) and the Hellenic Society of Theoretical and Applied Mechanics (HSTAM).

This year, *8th International Congress of the Serbian Society of Mechanics* received more than 150 high-quality research papers. Each paper was reviewed and ranked by at least 2 professors and scientists in the program and the scientific review committee. As a result of the strict review process and evaluation, the committee selected 120 research papers.

We must also say that the conference would certainly not have been so successful without the efforts of many people who were actively engaged in organization of such a major nationally and internationally recognized academic event. We give our special gratitude to the members of the program and scientific review committee as well as to all chairs, organizers and committee members for their dedication and support.

On behalf of the Organizing Committee, we wish you all a pleasant stay in Kragujevac and a productive conference.

Chairs:

Prof. Nenad Filipović, ***president of SSM, University of Kragujevac***
Prof. Miloš Kojić, ***Serbian Academy of Sciences and Arts***

Technical Program

Monday 28 June 2021

08:45 - 09:15	Opening Ceremony - Welcome speech: **Prof. Nenad Filipović**, President of SSM, Conference Co-Chair **Prof. Miloš Kojić**, full member of SASA, Conference Co-Chair **Nikola Dašić**, Major of Kragujevac City **Prof. Ivica Radović**, State Secretary, Ministry of Education, Science and Technological Development, Serbia **Prof. Dobrica Milovanović,** Dean of Faculty of Engineering, Kragujevac

09:15 - 09:45	Keynote speaker: Topic: **Analysis of a New Mixed Formulation for Hyperelasticity Using Kirchhoff Stress** **Prof. Ricardo Ruiz Baier**, Monash University, School of Mathematics, Clayton, Australia Chair: Hedrih A.

Session M.1A: 09:45-11:00

Biomechanics (part I)

Chairs: Kojić M., Geroski V.

M.1A.1 – Extension of our computational model for the left ventricle tissue to include hypertrophy – *Kojić M.*

M.1A.2 – Coupled Ohara-Rudy numerical model for heart electro-mechanics – *Geroski V., Milošević M., Milićević B., Simić V., Filipović N., Kojić M.*

M.1A.3 – Electromyography detection of muscle response in musculus quadriceps femoris of elite volleyball players on different exercises – *Radaković R., Peulić A., Kovač S., Simojlović M., Filipović N.*

Session M.1B: 09:45-11:00

Mechanics of Solid Bodies (part I)

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M.1B.2 – Size-effect modeling of Weibull Jc cumulative distribution function based on a scaling approach – *Mastilović S., Đorđević B., Sedmak A.*

M.1B.3 – Material parameters identification of concrete damage plasticity material model – *Rakić D., Bodić A., Milivojević N., Dunić V., Živković M.*

M.1B.4 – Using of gap element for contraction joints modeling in seismic analysis of concrete arch dams – *Živković M., Jović N., Pešić M., Rakić D., Milivojević N.*

M.1B.5 – Finite element analysis of effects of multiple defects on welded joint integrity – *Aranđelović M., Sedmak S., Jovičić R., Sedmak A., Radaković Z.*

11:00 - 11:30	Coffee Break

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Interdisciplinary and Multidisciplinary Problems (part I)

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M.2A.2 – Nonlinear landslide dynamics – *Kostić S., Vasović N.*

M.2A.3 – Computational mechanics – welding joint as a case study – *Jeremić L., Sedmak A., Sedmak S., Martić I.*

M.2A.4 – Experimental electrochemotherapy using novel design single needle device – *Cvetković A., Cvetković D., Milasinović D., Jovičić N., Miailović N., Nikolić D., Mitrović S., Filipović N.*

M.2A.5 – Cavitation diagrams for merchant ships using four blade b series propellers – *Veg M., Kalajdžić M.*

M.2A.6 – Microfluidic lab-on-chip system development for cell culture cultivation – *Milivojević N., Živanović M., Nikolić D., Jovanović Ž., Šeklić D, Nikolić M., Filipović N.*

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Mechanics of Solid Bodies (part II)

Chairs: Rakić D., Obradović A.

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M.2B.2 – Mass minimization of an AFG Timoshenko cantilever beam with a large body placed eccentrically at the beam end – *Obradović A., Mitrović Z., Zorić N.*

M.2B.3 – On concentrated surface loads and the flat punch contact problem in strain gradient elasticity – *Zisis T., Gourgiotis P., Georgiadis H.*

M.2B.4 – Geometric optimization of shaft transition zone based on stress-strain analysis of nature inspired design – *Atanasovska I., Momčilović D.*

M.2B.5 – A comparative analysis of fatigue behaviour between S355J2+N and Strenx 700 steel grade – *Živković M., Milovanović B., Dišić A., Jovičić G., Topalović M.*

M.2B.6 – Linear transient analysis of spatial curved Bernoulli – Euler beam using isogeometric approach – *Jočković M., Nefovska Danilović M.*

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NOISE INDUCED DYNAMICS OF EARTHQUAKE NUCLEATION MODEL

Srđan Kostić[1], Nebojša Vasović[2]

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Abstract

In present paper, we analyze the effect of seismic noise on the onset and dynamics of earthquake nucleation models. For this purpose, we examine the model of 100 (model 1) and 10 (model 2) all-to-all coupled units in a system of blocks interconnected by springs of the same coupling strength. It is assumed there is a time delay in a displacement of neighboring blocks, which qualitatively corresponds to the effect of friction state variable. For model 1, we analyze the effect of random (white) seismic noise, whose natural occurrence is confirmed by the previous investigation works. Effect of the noise is such that it induces the transition between different dynamical regimes, which are analogous to different states of earthquake fault motion: equilibrium state or steady sliding and aseismic creep. For model 2, we analyze the effect of colored noise which is also previously confirmed to occur in natural conditions. Results of the analysis indicate the occurrences of transitions between steady state regime, aseismic creep and seismogenic fault motion. For both models, deterministic variants of starting stochastic models are developed using mean-field approximation.

Key words: spring-block model, bifurcation, random noise, colored noise, mean-field approximation

1. Introduction

Research on earthquakes generally has two main directions. One direction is concerned with the study of seismic ground motion, which is further in connection with effect of earthquakes on the structural stability and human lives. Another direction is linked to the investigation of earthquake nucleation, i.e. the focus is on the study of conditions for earthquakes to occur in the Earth's crust. Although these two directions are not necessarily linked, it is clear that clear understanding of the mechanism of earthquake nucleation determines further investigation of seismic ground motion, i.e. its surface effects, especially considering the fact that only part of the energy being released during the earthquake nucleation is felt in a form of seismic ground motion. In present paper, authors are concerned with the conditions for earthquake nucleation, under the effect of seismic noise.

Increased interest for the analysis of earthquake nucleation dynamics started with the suggestion of Burridge-Knopoff model in 1967, which represents an array of interconnected blocks moving along the rough surface. Burridge and Knopoff [1] showed that such model exhibits the power-law behavior, i.e. relation between number and magnitude of recorded events nearly follows the macroseismic Guttenberg-Richter law. Moreover, results of their analysis indicated that the subsequent events, the so-called ‘’aftershocks’’ Omori-Utsu law. Apparently, such findings provided enough proof for future researchers to form a whole new research direction dealing with the properties of spring-block models "imitating" the dynamics of earthquake nucleation zone, including the work by Carlson and Langer [2], Ranjith and Rice [3], Galvanetto [4] and Erickson et al. ([5]. Besides the work of Burridge and Knopoff [1], another important research strongly influenced the further research on earthquake nucleation process – this is the work made by Dieterich [6] and subsequently Ruina [7], who investigated the role of friction in the dynamics of spring-block model and suggested the new rate – and state – dependent friction law, the so-called Dieterich-Ruina friction law. Further research on the role of friction provided significant results in this field [5,8,9].

Despite many published papers on the dynamics of spring-block model with different friction laws, only few of them included the analysis of the effect of seismic noise. One should note that existence of seismic noise is confirmed in the previous research of authors. In particular, Vasović et al. [10] indicated prevailing stochastic nature by surrogate data analysis of available GPS measurements of active fault movement. Also, Kostić et al. [11] confirmed the presence of autocorrelation in the background of seismic noise, using the measurement of real fault movement, and the recorded ground shaking before and after an earthquake.

In general, background seismic noise has small amplitudes and high frequencies, and its effect should not be significant to the dynamics of displacement and its derivatives (velocity and acceleration). However, it is known, e.g. from neural dynamics, that noise could induce transitions between different dynamical regimes [12]. Also, authors conducted some previous research on this topic. For example, Vasović et al. [10] shown for the first time that irregular mean displacement of a whole fault near the transition from equilibrium state to aseismic creep could occur either due to sole effect of seismic noise or under the impact of seismic noise in the presence of local and global attractor. Also, Kostić et al. [11] indicated that the effect of colored noise lies in the possibility of generating the seismic fault motion even for small values of correlation time.

From the seismological viewpoint, it is of particular interest to provide insight into the possibility that seismic noise could induce the onset of earthquake nucleation process. This possibility is of particular importance both for theory and practice, since this could mean that origin of earthquakes could not be only prescribed to large tectonic movements and the release of accumulated energy, but also the low-amplitude high-frequency noise could also make significant contribution to earthquake triggering.

In present paper, authors analyze effect of random and colored noise on the onset of earthquakes, i.e. authors determine the conditions which lead to onset of irregular oscillations.

2. Earthquake model with random seismic noise

The first earthquake nucleation model is defined as a spring-block model composed of 100 all-to-all coupled units, whose motion is described by the following system of stochastic delay differential equations:

$$dU_{1i}(t) = U_{2i}(t)dt$$
$$dU_{2i}(t) = \left\{-U_{1i}(t) + \Phi\left(U_{2_i} + \nu\right) - \Phi(\nu) + \frac{K}{N}\sum_{j=1}^{N}\left[U_{1j}(t-\tau) - U_{1i}(t)\right]\right\}dt + \sqrt{2D}dW_i \quad (1)$$

where K is spring constant (coupling strength, assuming to be equal to all the blocks in the system), Φ is the friction force ($\Phi(U) = \mu_0 + a\, ln(U)$), U_{1i} and U_{2i} represent displacement and velocity of the i-th block, respectively, τ is time delay (equal for all blocks) and ν is a nondimensional pulling background velocity, μ_0 is a steady state friction and a represent a material property which depends on different temperature and pressure conditions Term $(2D)^{1/2}dWi$ represent stochastic increments of independent Wiener process:

$$E\left(dW_i\right)=0,\, E(dW_i dW_j)=\delta_{i,j}dt\,, \tag{2}$$

where E () denotes the expectation over many realizations of the stochastic process and D is intensity of additive local noise. Assumed friction in (1) resembles of the friction force already proposed by Scholz [13], where state dependent term, which is incorporated by introducing the time delay τ .

From the viewpoint of nonlinear dynamics, in order to be able to analyze dynamics of system (1), we derive its deterministic approximation using the method of mean-field approximation, based on a set of approximations that replace a many component system by a simpler system described by a small number of average macroscopic properties.

By deriving the Taylor expansion of $\Phi(U_2(t)+\nu)$ in the vicinity of the mean values $(\langle U_1\rangle,\langle U_2\rangle)=\left(\lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}U_{1i}(t)\,,\lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}U_{2i}(t)\right)=\left(m_{U_1},m_{U_2}\right)$ system (1) in the main text, in the thermodynamic limit, for $N\to\infty$, becomes:

$$dU_{1_i}(t)=U_{2i}(t)dt$$
$$dU_{2i}(t)=\{-U_{1i}(t)+\Phi(m_{U_2}+\nu)-\Phi(\nu)+\frac{1}{1!}[\Phi'(m_{U_2}+\nu)][U_2(t)-m_{U_2}]+$$
$$+\frac{1}{2!}[\Phi''(m_{U_2}+\nu)][U_2(t)-m_{U_2}]^2+\frac{1}{3!}[\Phi'''(m_{U_2}+\nu)][U_2(t)-m_{U_2}]^3+$$
$$+\frac{1}{4!}[\Phi^{(4)}(m_{U_2}+\nu)][U_2(t)-m_{U_2}]^4\;+K[m_{U_1}(t-\tau)-m_{U_1}(t)]\}dt+\sqrt{2D}dW_i. \tag{3}$$

Following the procedure from Burić et al. [14], starting system (3) of N SDDEs is reduced to the system of only five deterministic DDEs for the global variables and global centered moments: $m_{U_1}(t)=\langle U_1(t)\rangle,\ m_{U_2}(t)=\langle U_2(t)\rangle,\ s_{U_1}(t)=\langle n_{U_1}^2(t)\rangle,\ s_{U_2}(t)=\langle n_{U_2}^2(t)\rangle,\ s_{U_1U_2}(t)=\langle n_{U_2}\cdot n_{U_1}\rangle$ where $n_{U_j}(t)=m_{U_j}(t)-U_{ij}(t)$, j=1,2.

Within present research, method of mean-field approximation gave the final approximated model (1) in the following form:

$$\dot{m}_{U_1}(t)=m_{U_2}(t),$$
$$\dot{m}_{U_2}(t)=-m_{U_1}(t)+a\ln\nu-a\ln\left(m_{U_2}+\nu\right)+\frac{a}{2}\frac{1}{\left(m_{U_2}+\nu\right)^2}s_{U_2}+\frac{a}{4}\frac{1}{\left(m_{U_2}+\nu\right)^4}3s_{U_2}^2\;+$$
$$+K\left[m_{U_1}(t-\tau)-m_{U_1}(t)\right],$$
$$\frac{1}{2}\dot{s}_{U_1}(t)=s_{U_1U_2}$$
$$\frac{1}{2}\dot{s}_{U_2}(t)=s_{U_2}\left[-\frac{a}{\left(m_{U_2}+\nu\right)}-\frac{a}{\left(m_{U_2}+\nu\right)^3}s_{U_2}\right]-(K+1)s_{U_1U_2}+D,$$
$$\dot{s}_{U_1U_2}=-s_{U_1U_2}\left[\frac{a}{(m_{U_2}+\nu)}+\frac{a}{(m_{U_2}+\nu)^3}s_{U_2}\right]-(K+1)s_{U_1}+s_{U_2}(t) \tag{4}$$

Local bifurcation analysis of the approximated model (4) is conducted numerically using DDE-BIFTOOL in Matlab. Results obtained indicate a transition from equilibrium state to periodic oscillations for certain parameter values, as it is shown in Fig. 1a.

From the dynamicsl viewpoint, equilibrium state for the starting model (1) is represented by small fluctuations around the constant zero value of a mean-field approximated displacement for a mean-fied model (4). Equilibrium state and periodic oscillations are clearly captured only in K –τ diagram (Figure 1b). Bifurcation curves in diagrams where parameters a, τ and K are given as function of D, are captured only for the mean-field approximated model (4). When bifurcation curve is crossed, oscillation frequencies are the same for both the starting model (1) and approximated system (4), while amplitude could be slightly different due to effect of the introduced random seismic noise in the stochastic model (1).

From the seismological viewpoint, observed dynamical regimes could be assigned to different regimes of aseismic motion. In particular, equilibrium state corresponds to the state when there is no movement along the fault or steady sliding along the fault in the original system (1), while the aseismic creep along the fault could be described by the periodic oscillations of small amplitude.

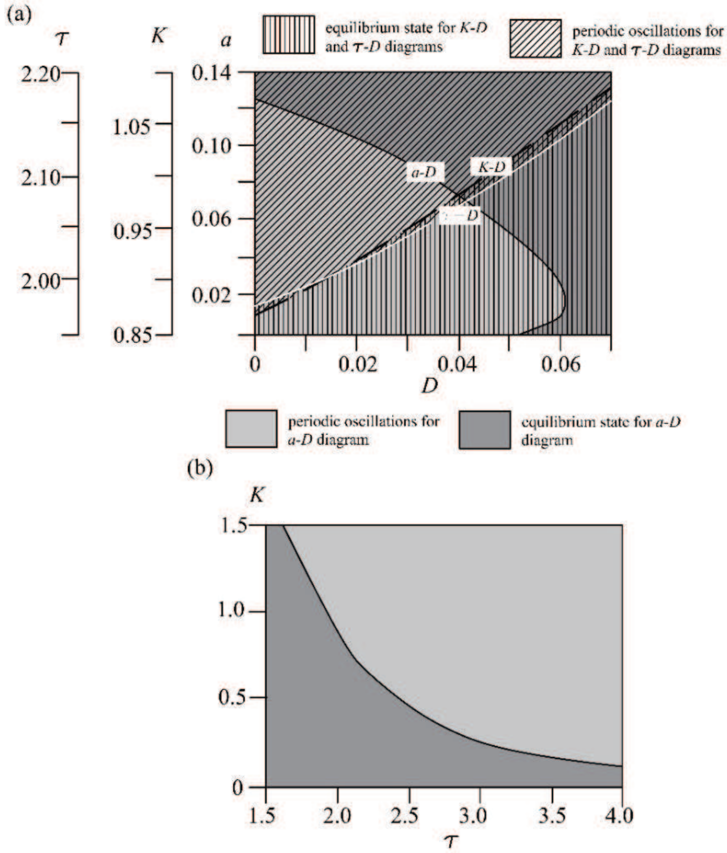

Fig. 1. Bifurcation diagrams *a-D, K-D, K-τ* and *τ-D* admitting equilibrium state or periodic oscillations of the mean-field approximated model (4). For a given parameter domain, other parameters are held constant at the following values: *K=0.9, v=1.2, D=0.001, τ=2, a=0.1*.

3. Earthquake model with colored additive seismic noise

The second model of earthquake fault motion is a model of all-to-all coupled spring-slider system with 10 units, whose dynamics is described by the following set of stochastic delay differential equations:

$$\dot{x}_i(t) = y_i(t)$$
$$\dot{y}(t) = -x_i(t) + \Phi(y_i + v) - \Phi(v) + \frac{K}{N}\Big(x_j(t-\tau) - x_i(t)\Big) + Z_i(t)$$
$$dZ_i(t) = -\frac{Z_i}{\varepsilon}dt + \sqrt{\frac{2D}{\varepsilon}}dW_i \tag{5}$$

where x_i and y_i represent displacement and velocity of the i-th block, respectively, while parameters are the same as for model (1). $Z_i(t)$ is an Ornstein-Uhlenbeck process, with ε as the noise correlation time and D as the intensity of noise. Colored noise generated by Ornstein-Uhlenbeck process with this parametrization is referred to as power-limited colored noise, since the total power of the noise (the integral over the spectral density of the process) is conserved upon varying the noise correlation time. Friction force Φ is assumed to be only rate-dependent: $\Phi(V) = -(\mu_0 + a\,ln(V))$ where V is the general notion for the friction arguments in (5).
By deriving the Taylor expansion of $\Phi(y_i(t)+v)$ in the vicinity of the mean values

$(\langle x_i\rangle, \langle y_i\rangle, \langle z_i\rangle) = (m_x, m_y, m_z) = \left(\lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} x_i(t), \lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} y_i(t), \lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} z_i(t)\right)$,
system (5) becomes:

$$\dot{x}_i(t) = y_i(t)$$

$$\begin{aligned}\dot{y}(t) = &-x_i(t) + \Phi(m_y + v) - \Phi(v) + \frac{1}{1!}[\Phi'(m_y + v)][y_i(t) - m_y] \\ &+ \frac{1}{2!}[\Phi''(m_y + v)][y_i(t) - m_y]^2 + \frac{1}{3!}[\Phi'''(m_y + v)][y_i(t) - m_y]^3 \\ &+ \frac{1}{4!}[\Phi^{(4)}(m_y + v)][y_i(t) - m_y]^4 + K[m_x(t-\tau) - x_i(t)] + Z_i(t)\end{aligned}$$

$$dZ_i(t) = -\frac{Z_i(t)}{\varepsilon}dt + \sqrt{\frac{2D}{\varepsilon^2}}dW_i \tag{6}$$

The following deviations from the mean-field are introduced: $\langle x(t)\rangle = \lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} x_i(t), \langle y(t)\rangle = \lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} y_i(t), \langle z(t)\rangle = \lim\limits_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} z_i(t)$, for each element $n_x(t) = \langle x(t)\rangle - x_i(t), n_y(t) = \langle y(t)\rangle - y_i(t), n_z(t) = \langle z(t)\rangle - z_i(t)$.
In further calculation, we use the following notation for the first and the second order cummulants:
- The means: $m_x(t) = \langle x(t)\rangle, m_x(t-\tau) = \langle x(t-\tau)\rangle, m_y(t) = \langle y(t)\rangle, m_z(t) = \langle z(t)\rangle$,
- The mean square deviations: $s_x(t) = \langle n_x^2(t)\rangle, s_y(t) = \langle n_y^2(t)\rangle, s_z(t) = \langle n_z^2(t)\rangle$,
- The cross-cummulants: : $U_{xy}(t) = \langle n_x n_y\rangle, U_{xz}(t) = \langle n_x n_z\rangle, U_{yz}(t) = \langle n_y n_z\rangle$.

Following the procedure from Kostić et al. (2019) we obtain the foolowing mean-field approximation of the starting model (5):

$$\dot{m}_x = m_y$$

$$\begin{aligned}\dot{m}_y = &-m_x - a\,ln(m_y + v) + a\,ln(v) + \frac{1}{2}\frac{a}{(m_y + v)^2}s_y + \frac{3}{4}\frac{a}{(m_y + v)^4}{s_y}^2 \\ &+ K(m_x(t-\tau) - m_x) + m_z\end{aligned}$$

$$\dot{m}_z = -\frac{1}{\varepsilon}m_z$$

$$\frac{1}{2}\dot{s}_x = U_{xy}$$

$$\frac{1}{2}\dot{s}_y = s_y\left[-\frac{a}{m_y + v} - \frac{a}{(m_y + v)^3}s_y\right] - (K+1)U_{xy} + U_{yz}$$

$$\dot{U}_{xy} = U_{xy}\left[-\frac{a}{m_y + v} - \frac{a}{(m_y + v)^3}s_y\right] - (K+1)s_x + s_y + U_{xz}$$

$$\dot{U}_{xz} = U_{yz} - \frac{1}{\varepsilon}U_{xz}$$

$$\dot{U}_{yz} = -U_{xz} - \frac{a}{m_y+v}U_{yz} - \frac{a}{(m_y+v)^3}s_y U_{yz} - KU_{xz} + D - \frac{1}{\varepsilon}U_{yz} \tag{7}$$

Results obtained indicate the existence of three dynamical regimes (Fig. 2): (1) equilibrium state as steady stationary movement (corresponding to the steady regime of fault motion); (2) small-amplitude regular periodic oscillations (corresponding to the creep regime of fault motion); (3) high-amplitude irregular oscillations (corresponding to the seismogenic fault motion). From Fig. 2 one could identify the effect of correlation time ε on the dynamics of mean-filed model (2). In particular, second bifurcation curve vanishes with the increase of correlation time, i.e. there are no high-amplitude oscillations, which could indicate that degree of autocorrelation of background seismic noise could directly determine the type of transition from creep regime of fault dynamics to low-amplitude oscillations or to high-amplitude irregular oscillations, whose amplitude progressively increases. This could be interpreted as the onset of the fault motion which produces the seismic waves responsible for surface soil shaking.

On the other hand, the coupling strength K, impacts both time delay τ and friction a, and excludes the possibility of the occurrence of seismogenic fault motion. In particular, for higher values of K transition from equilibrium state to creep regime is possible even for higher values of friction a, meaning that the stronger interrelations between different patches of fault also induce the stronger role of friction.

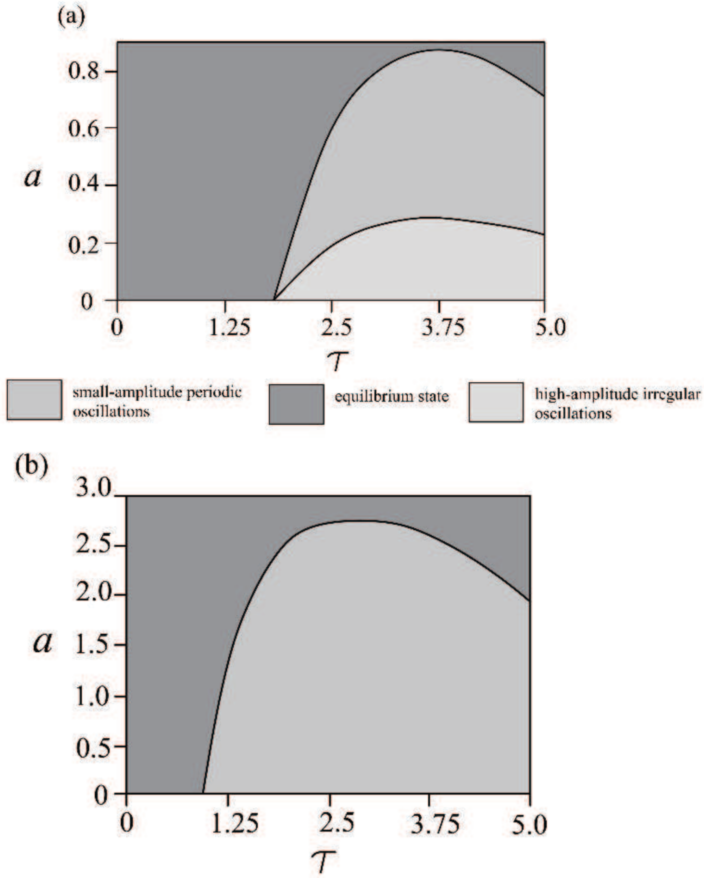

Fig. 2. Andronov-Hopf bifurcation diagrams, displaying interaction of friction a and time delay τ, for different values of correlation time ε. While friction and delay are being varied, other parameters are being held constant for the mean-field model (7) in equilibrium state: $\mu_0 = 0.1$, $K = 1$ *(a)*, $K=5$ *(b)*, $D = 0.001$, $v = 1.2$, ε = 0.005-5. For ε=5.0 no high-amplitude irregular oscillations occur.

For lower values of time delay and higher values of K, there are no conditions for seismogenic motion to occur. It could be seen from Fig. 2 that high-amplitude irregular oscillations occur for higher values of time delay, i.e. $\tau > 5$. From the practical viewpoint, this means that the higher delay in interaction between the neighboring patches of fault –the more likely is to expect the onset of seismogenic fault motion.

4. Conclusions

In present paper, we show that earthquake nucleation is possible solely under the effect of random or colored seismic noise, which represents rather uncommon finding, since small-amplitude noise should not have any significant effect on the onset of regular/irregular oscillations – displacements along the fault. However, it is shown that random noise could trigger the transition from equilibrium or steady state sliding to aseismic creep, while colored noise could even lead to seismogenic fault motion. The effect of correlation time ε on earthquake nucleation process is rather interesting, since the increase of ε drives away from the conditions that lead to earthquake nucleation. The same effect is captured for the higher value of coupling strength K. On the other hand, the increase of the included time delay in positions of the neighboring blocks leads to the occurrence of complex dynamical behavior. It seems that without the delay in interaction, or with the small values of delay, the whole fault acts as a unique block, i.e. the fault patches are locked, preventing the irregular seismogenic motion to occur.

One should note that this research represents only the starting point of the extensive work in this direction. Further research on this topic could include the analysis of the more complex setup of the spring-block model, where blocks are only coupled to each other, or the case where coupling strength between blocks decreases with their mutual distance. Also, one could further investigate the case where blocks are not only connected along a single array, but instead are connected in a certain spatial manner. All these cases certainly lead to more realistic description of earthquake nucleation process.

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