



Multidimensional Approaches to Calculation of Design Floods at Confluences—PROIL Model and Copulas

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Abstract

In confluence areas, where river flow slowdowns can be substantial, the selection of design flows for the purpose of designing a flood control system directly depends on flood waves at the mainstem and its tributaries. The aim of this paper is twofold: to elaborate a procedure that will define the flood exceedance probability in the analyzed area within a multidimensional probability space and to develop a methodology for defining flood coincidence in the mainstem reach with its main tributaries and thus determine design flows in order to design the flood control system in the confluence area. Mathematical models are based on (1) two-dimensional probability distribution (PROIL model) and (2) the copula function (Archimedean class of copulas), and fitted to a practical application in a two-dimensional probability distribution space. In case of random variables, simultaneous quantitative characteristics of flood wave (water flow) hydrographs for the mainstem and a tributary are considered. The paper discusses the specific reach of the Danube, from the Hungarian-Serbian border to the city of Smederevo, which encompasses three significant tributaries—the Drava, the Tisa, and the Sava.

Keywords Confluence · PROIL model · Archimedean copulas · Design flood level · Flood control system

1 Introduction

Planning and design of flood adaptation and control measures relies on a probabilistic estimation of flood wave parameters. The definition of design flow on rivers is necessary for a variety of engineering purposes [1], the most prominent of which is the determination of design

water levels in order to design a flood control system, which includes structures such as embankments.

The established practice is to use probabilistic analysis and determine the theoretical values of maximum annual flows measured at the nearest gauging station. However, the complexity of floods has imposed the need to approach this event as a multidimensional phenomenon [2]. If river reaches include tributaries, such an approach cannot be justified, because at least two flows (mainstem and tributary) have to be considered. Accordingly, the complexity of flood occurrence and its assessment require the linking of marginal distributions of multiple variables in order to define a unique distribution law that describes the flood [3].

The first hydrological study involving two-dimensional analysis was published by Matalas and Langbein in 1962 [4]. In 1983, according to Ashkar and Aucoin [4], Stedinger proposed the use of generalized regression when calculating design floods for designing, which takes into account the regional dependence of hydrological measurements. According to [1], the issue of two-dimensional probability distribution density was discussed by Morris and Calise and Raynal and Salas in 1987, while the notion of a simultaneous flood on two watercourses first appeared in [5].

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Near the end of the 20th century, Marshall and Olkin [6] highlighted the increased interest in multiple probability distributions (bivariate, two-dimensional, with marginal probabilities as distribution parameters).

Calculation of distribution quantiles in a multidimensional space enables different combinations of variables that yield the same risk [7]. By observing complex phenomena, the following authors applied the following probability distributions in a multidimensional space: double Gaussian distribution and exponential distribution [8], double Gumbel or mixed Gumbel distribution [9], double normal distribution [10–12], double lognormal distribution [13], and double gamma distribution [14]. A comprehensive listing of multidimensional probability distributions can be found in [15].

In most studies, multiple probabilities distribution was obtained based on the following assumptions: variables follow the same probability distribution function, the joint distribution of two variables is normal or transformed to normal, and variables are considered to be independent.

From the early 2000s, copulas became a popular tool for multidimensional analyses in hydrology [16] and for creating multidimensional probability distributions without relying on the aforementioned assumptions.

Copulas are used for modelling in the following fields: (1) droughts [2, 17–20], (2) precipitation [21–26], (3) groundwater [27], (4) dam overtopping risk determination [28], (5) dependence between flow and concentration of suspended sediment load [29], (6) probabilistic analyses of floods [30–36], (7) flood wave coincidences [1, 37, 38], and (8) the impact of climate change on precipitation extremes [39, 40].

A comprehensive review and description of copulas can be found in [16] and [41].

The following section defines the area under research, specifically the reach of the Danube from Bezdan gauging

station to Smederevo gauging station, which includes three significant tributaries—the Drava, Tisa, and Sava rivers. Series of maximum annual flows and other flows were created according to the available data from the Republic Hydro Meteorological Service of Serbia (RHMS) for the period between 1931 and 2014. Section 2 describes multidimensionality at a confluence of two rivers, the construction of multidimensional probability distribution, the PROIL model, which is based on normal bivariate distribution, and Archimedean copulas. It explains the meaning of flood coincidence of the mainstem and its tributary in terms of determining design flows in the confluence area in order to design the flood control system. The results are presented and discussed in Sect. 3, while the conclusions are presented in Sect. 4.

2 Materials and Methods

2.1 Study Area and Input Data

Methods and procedures defined in the following subsections were applied on the reach of the Danube from its entry point into Serbia to the city of Smederevo (Fig. 1).

A flood at river reaches that include tributaries should be viewed as a complex event that can only be described in a multidimensional space. Thus, it is often the case that the significant tributaries form a flood wave on the mainstem [37]. Flood determination at confluences with the major tributaries of the Danube was based on the data of original series of maximum annual flows (Q_{max}) and on the corresponding mean daily flows ($Q_{dy} - Q_{corr}$). The overview of selected profiles (GS) and available gauging periods is given in Table 1.

Fig. 1 Combinations of GS by nodes for the Danube reach from its entry into Serbia to the city of Smederevo

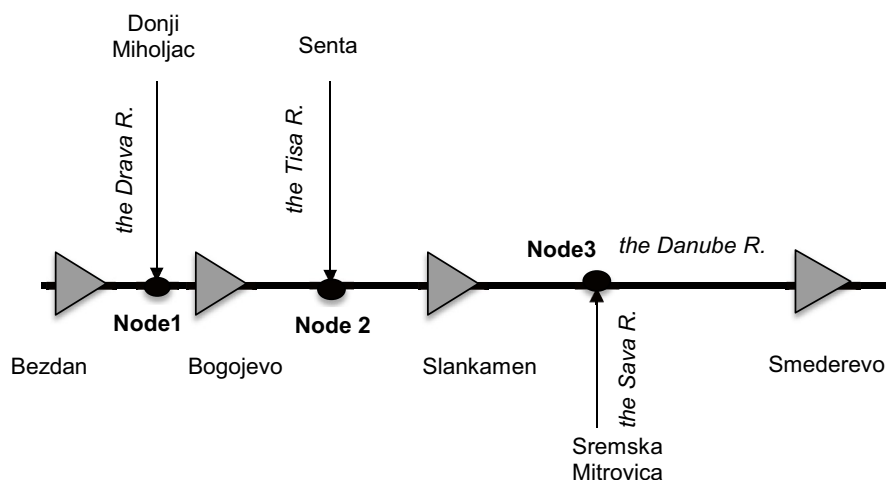


Table 1 Overview of gauging stations and available flow data on the Danube reach from its entry into Serbia to the city of Smederevo, including the tributaries

No.	Station number	River	Profile (GS)	Distance from the confluence (km)	Catchment area (km ²)	Periods for analysis	
						Q_{corr} (m ³ /s)	Q_{max} (m ³ /s)
1	42010	Dunav	Bezdan (BEZ)	1425.59	210,250	1931–2014	1950–2014
2	42020	Dunav	Bogojevo (BOG)	1367.25	251,593	1931–2014	1950–2014
3	42040	Dunav	Slankamen (SLAN)	1216.02	254,961	1931–2014	1946–2014
4	42055	Dunav	Smederevo (SMED)	1116.23	525,820 (525,009)	1931–2014	1946–2014
5	5150	Drava	Donji Miholjac (DM)	80.50	37,142	1931–2014	1952–1989
6	44020	Tisa	Senta (SENT)	123.50	141,715	1931–2014	1946–2014
7	45090	Sava	Sremska Mitrovica (SM)	139.24	87,996	1931–2014	1946–2014

2.2 The PROIL Model

The theory of multiple-coincidence of flood waves in river reaches with tributaries relies on the practical application of multiple probability distribution functions and their conditional probability. Simultaneous quantitative characteristics of the flood wave hydrograph on the mainstem or on one or more significant tributaries are taken as representative or random variables. This study concerns the issue of simultaneous occurrence of flood waves on the mainstem and the tributaries, as well as the basic parameter of flood hydrograph related to the flood control system—the flood wave flow.

In accordance with statistical theory, the density function of a two-dimensional random variable is defined as:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{-\frac{x-\mu}{\sigma} \left\{ -\frac{1}{2(1-\rho^2)} \left[\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right] \right\}}, \tag{1}$$

where:

X and Y random variables (flood wave characteristics of the mainstem and its tributary)

x and y simultaneous realization of X and Y random variables

μ_x and μ_y expected values of X and Y random variables

σ_x and σ_y standard deviations of X and Y random variables

ρ correlation coefficient between X and Y random variables

$$\rho = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \mu_x)(y - \mu_y) \cdot f(x, y) dx dy}{\sqrt{\int_{-\infty}^{\infty} (x - \mu_x) f(x) dx \cdot \int_{-\infty}^{\infty} (y - \mu_y) f(y) dy}} \tag{2}$$

For the density function of a two-dimensional random variable $f(x,y)$, marginal densities $f(x, \bullet)$ and $f(\bullet,y)$ are defined as:

$$f(x, \bullet) = \int_{y=-\infty}^{y=\infty} f(x, y) dy, \tag{3}$$

$$f(\bullet, y) = \int_{x=-\infty}^{x=\infty} f(x, y) dx. \tag{4}$$

The marginal cumulative probability functions are determined in the following way:

$$F(x, \bullet) = \int_{t=-\infty}^{t=x} f(t, \bullet) dt \tag{5}$$

and

$$F(\bullet, y) = \int_{z=-\infty}^{z=y} f(\bullet, z) dz. \tag{6}$$

The cumulative probability function $F(x,y)$ is defined as follows:

$$F(x, y) = P[X \leq x \cap Y \leq y] = \int_{t=-\infty}^{t=x} \int_{z=-\infty}^{z=y} f(t, z) dt dz. \tag{7}$$

The cumulative probability function $\Phi(x,y)$ can be calculated using the following expression:

$$\begin{aligned} \Phi(x, y) &= \int_{t=x}^{t=+\infty} \int_{z=y}^{z=+\infty} f(t, z) dt dz \\ &= P[X > x \cap Y > y] \\ &= 1 - P[X > x \cup Y > y] \\ &= 1 - F(x, \bullet) - F(\bullet, y) + F(x, y). \end{aligned} \tag{8}$$

More details about the theoretical background for defining bivariate distribution functions and the grapho-analytical procedure [42] can be found in the literature [10, 11].

2.3 Copulas

Copulas are tools for determining the dependence of several random variables. They were first introduced by Sklar in 1959, and their main purpose is the description of the links of several random variables [43]. The mathematical model is based on a function that joins several probability distribution functions into a single joint distribution (Fig. 2). Let there be a pair of random variables (X, Y) and let their cumulative distribution functions be $F_X(x)=P[X\leq x]$ of X and $G_Y(y)=P[Y\leq y]$ of Y . Their joint cumulative distribution function will then be $H_{X,Y}(x, y)$. Figure 2 shows that every pair (x, y) corresponds to a point of a unit rectangle $[0,1] \times [0,1]$. Hence, it also corresponds to number $H_{X,Y}(x, y)$ over the interval $[0,1]$ [31].

All of the above applies only if the one-dimensional marginal distributions are uniform and non-decreasing functions.

Construction of copulas is closely connected with measures of dependence. The Pearson correlation coefficient is not a suitable measure of dependence for multidimensional probability distributions that are not normal. Therefore, a covariance will not provide information about the dependence between two random variables. The most widespread measures of dependence are Kendall’s tau coefficient τ_k and Spearman’s rank correlation coefficient ρ_s .

A widely used class of copulas, the so-called Archimedean copulas (Cs), was used in this study. A parameter of C is computed based on Kendall’s tau value.

Kendall’s correlation coefficient is a non-parametric measure of dependence based on the number of concordances or discordances in a measured pair sample.

Table 2 Types of Archimedean copulas

Copula	Parameter	$C(u,v,\theta)$	Eq. number
Gumbel (Gm)	$\theta \geq 1$	$\exp \left\{ -\left[(-\ln u)^\theta + (-\ln v)^\theta \right]^{\frac{1}{\theta}} \right\}$	11
Clayton (Cl)	$\theta > 0$	$\left[\max \{ u^{-\theta} + v^{-\theta} - 1, 0 \} \right]^{-\frac{1}{\theta}}$	12
Frank (Fr)	$\theta \in \mathfrak{R} / \{0\}$	$-\frac{1}{\theta} \log \left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)} \right]$	13

When pairs of random variables vary together, they are concordant; otherwise, they are discordant [31].

For a pair (X, Y) with a cumulative distribution function $H_{x,y}(x,y)$, τ_k can be defined as the difference between concordance and discordance probabilities for two independent pairs, (X_1, Y_1) and (X_2, Y_2) , each with the distribution $H_{x,y}(x, y)$ [44]:

$$\begin{aligned} \tau_{x,y} &= P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0] = \\ &= 2P[(X_1 - X_2)(Y_1 - Y_2) > 0] - 1 = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1, \end{aligned} \tag{9}$$

where

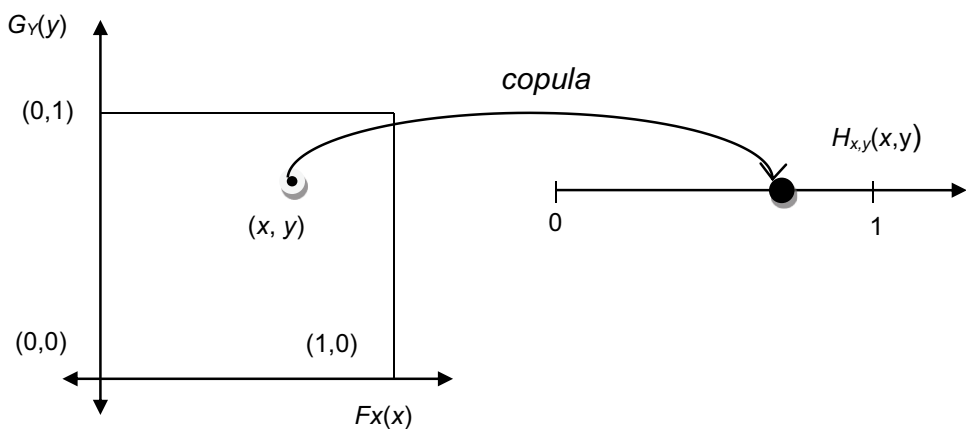
$$\begin{aligned} P\{\text{concordance}\} &= P[(X_1 - X_2)(Y_1 - Y_2) > 0], \\ P\{\text{discordance}\} &= P[(X_1 - X_2)(Y_1 - Y_2) < 0]. \end{aligned} \tag{10}$$

Table 2 shows three Archimedean copulas—Gumbel (Gm), Clayton (Cl), and Frank (Fr). All of them are one-parametric, with parameter θ [16].

For more information on the characteristics and construction of the Archimedean copulas used in this study, please see Chapter 4 in [16] and Appendix C in [41].

In risk theory, it is usually necessary to assess the possibility of exceeding the projected lifetime of a system.

Fig. 2 Graphic interpretation of COPULA function



This is determined using the probability of exceeding the time x , as defined by the survival function. For a pair of random variables (X, Y) with a joint probability distribution H , the joint survival function is

$$\bar{H}(x, y) = P[X > x, Y > y]. \tag{14}$$

In accordance with Sklar’s theorem [16], let copula C of random variables X and Y be

$$\begin{aligned} \bar{H}(x, y) &= 1 - F(x) - G(y) + H(x, y) \\ &= \bar{F}(x) + \bar{G}(y) - 1 + C(F(x), G(y)) = \\ &= \bar{F}(x) + \bar{G}(y) - 1 + C(1 - \bar{F}(x), 1 - \bar{G}(y)). \end{aligned} \tag{15}$$

Accordingly, function $\hat{C}: I^2 \rightarrow I$ can be defined as:

$$\hat{C}(u, v) = u + v - 1 + C(1 - u, 1 - v) \tag{16}$$

$$\Rightarrow \bar{H}(x, y) = \hat{C}(\bar{F}(x), \bar{G}(y)). \tag{17}$$

The quality of fitting a copula to the sample was evaluated based on the statistical test in [45] first proposed by Genest, Remillard, and Beaudoin [46]. The test involves a parametric bootstrap procedure. For every copula, n random independent samples are generated and a synthetic series of observations is determined and then compared to the synthetic series generated using the Monte Carlo simulation based on measured data. The test relies on measurements and does not depend on the selected marginal probability. To determine statistical significance, the p value is used, calculated based on the Cramer-von Mises statistic S_n [45] and compared to significance level α .

2.4 Flood Coincidence on the Mainstem and a Tributary

For the purpose of analyzing flood waves in the confluence area, flow was adopted as the main parameter of the flood hydrograph. Coincidence of flood occurrence is based on the analysis of the nearest hydrological profiles on the mainstem upstream and downstream from the tributary. The adopted properties of flood wave coincidence are shown in Figs. 3 and 4.

Symbols in Figs. 3 and 4 denote the following:

QIN_{max} , $QOUT_{max}$, QTR_{max} —maximum annual mainstem or tributary flow at the entry, exit, or tributary profile of the given river reach;

QIN_{corr1} , QTR_{corr2} —corresponding mainstem or tributary flow at the entry or tributary profile at the moment of

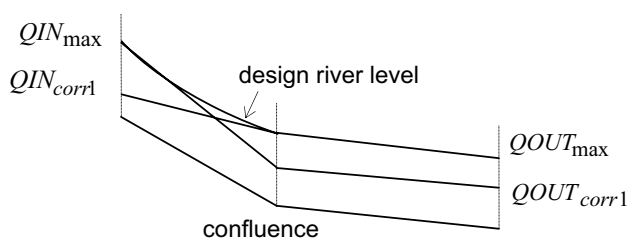


Fig. 3 Schematic of the mainstem reach that includes the confluence

maximum annual flow at the exit profile on the given mainstem reach;

$QOUT_{corr1}$, QTR_{corr1} —corresponding mainstem or tributary flow at the exit or tributary profile at the moment of maximum annual flow at the entry profile on the given mainstem reach;

QIN_{corr2} , $QOUT_{corr2}$ —corresponding mainstem flow at the entry or exit profile at the moment of maximum annual tributary flow.

The variables described represent the input for the PROIL model and for the copula procedure. The established dependences are used to define points 1, 2, and *, which form the basis for possible design flows to be used for designing the flood control system (Figs. 5 and 6).

Points 1 and 2 have the following coordinates:

Case 1 (Fig. 5):

$$\begin{aligned} \text{point 1 } &(QOUT_{corr1}^1 : QIN_{max}^1 (QTR_{max}^1))_p, \\ \text{point 2 } &(QOUT_{corr2}^2 : QIN_{max}^2 (QTR_{max}^2))_p. \end{aligned}$$

Case 2 (Fig. 6):

$$\begin{aligned} \text{point 1 } &(QIN_{corr1}^1 (QTR_{corr1}^1) : QOUT_{max,p}^1)_p, \\ \text{point 2 } &(QIN_{corr2}^2 (QTR_{corr2}^2) : QOUT_{max,p}^2)_p. \end{aligned}$$

The coincidence line (Figs. 5 and 6) can be used to read the most likely event (*), Eq. 18, with coordinates $(QIN_{corr}^* (QTR_{corr}^*) : QOUT_{max,p}^*)_p$ and $(QOUT_{max,p}^* : QIN_{corr}^* (QTR_{corr}^*))_p$ for both cases, respectively [1]:

$$(x^*, y^*) = \arg \max F_{X,Y}(F_X^{-1}(x), F_Y^{-1}(y)). \tag{18}$$

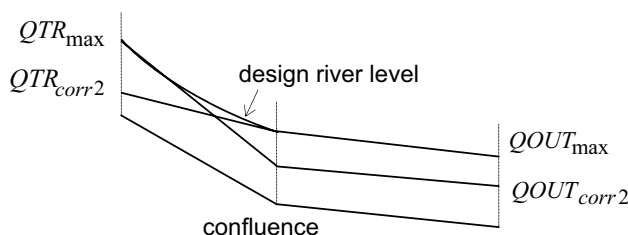


Fig. 4 Schematic of the mainstem reach downstream from the confluence

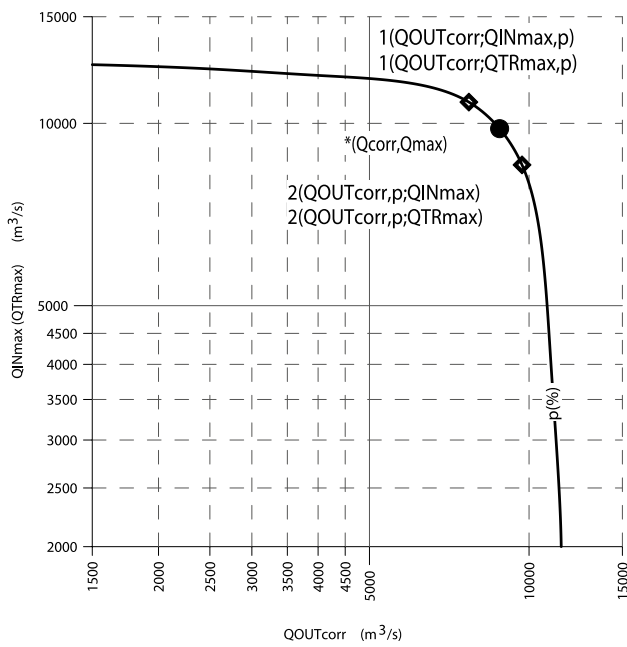


Fig. 5 Coincidence of the mainstem or tributary Q_{max} flow upstream from the confluence and of the Q_{corr} at the mainstem exit profile for the given exceedance probability

3 Results and Discussion

The first step in the algorithm for determining the most likely combinations of variables in a multidimensional space is to

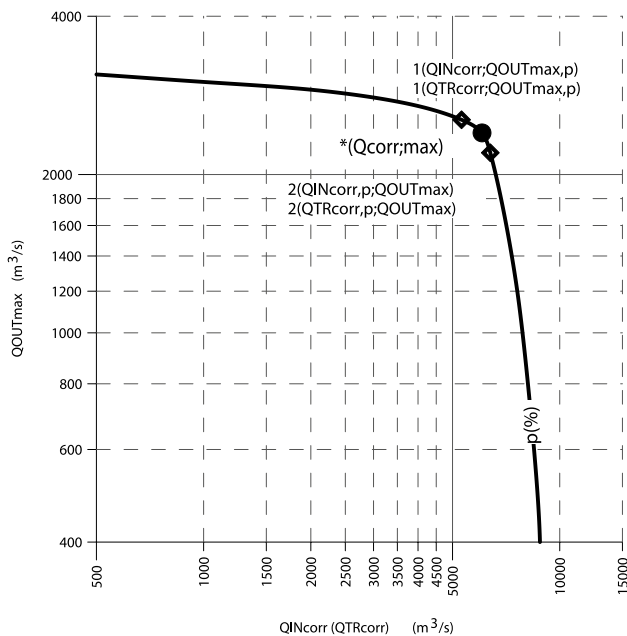


Fig. 6 Coincidence of the Q_{max} flow downstream from the confluence and of the Q_{corr} mainstem or tributary flow upstream from the confluence for the given exceedance probability

establish the marginal (“simple”) probability distributions. In the hydrological practice, distribution parameters were most often estimated using the method of moments. On the other hand, the maximum likelihood estimation (MLE), is associated with robust calculations, but it yields estimates that are more efficient for large samples than with any other method [47]. Marginal distribution parameters in the P model were estimated using the method of moments, whereas Cs were constructed using the MLE. The adopted marginal values for a selected exceedance probability $p = 1\%$, which are best fitted to the data according to tests, are shown in Appendix 1.

The main task of a flood control system design in confluence areas is an economically optimal design of all flood control facilities. Most often, these are river embankments, which spread beyond the river interaction zone during flood periods, for instance, from entrance gauging profiles on the mainstem and a tributary up to the exit profile on the mainstem. Calculation results are illustratively shown for node 1 in Figs. 7, 8, 9, 10, 11, and 12. The graphs contain isolines of probabilities of occurrence (density functions), lines of exceedance probabilities (distribution functions), and empirical points.

As described in Sect. 2.3, the construction of copulas is associated with Kendall’s tau coefficient τ_k measure of dependence, which is shown in Appendix 2 for each analyzed combination. The table also shows the copula parameter θ for every selected C according to the goodness of fit test, with the p value (Sect. 2.3). The survival copulas are illustratively shown in Figs. 13, 14, 15, 16, 17, and 18 for the determined node 1 combinations.

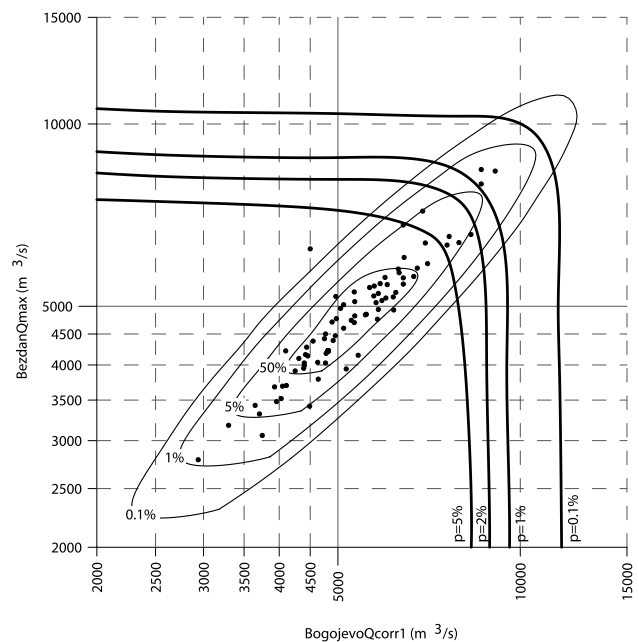


Fig. 7 Coincidence Q_{max} at Bezdán GS and Q_{corr1} at Bogojevo GS on the Danube

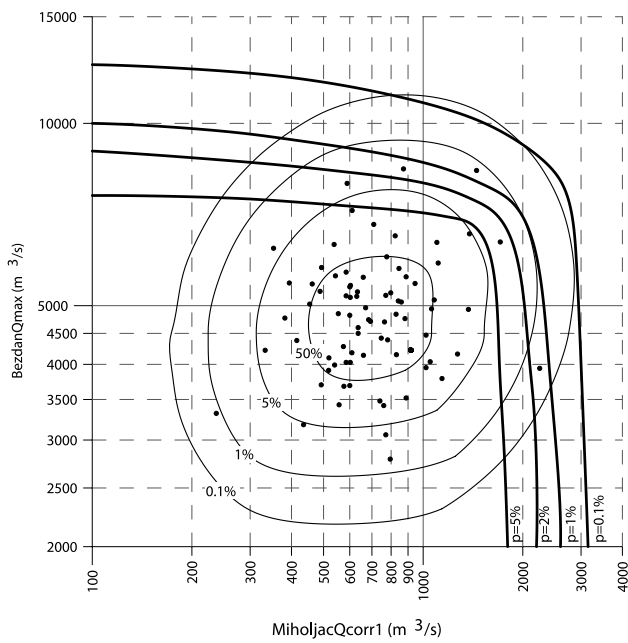


Fig. 8 Coincidence Q_{max} at Bezdán GS on the Danube and Q_{corr1} at D. Miholjac GS on the Drava

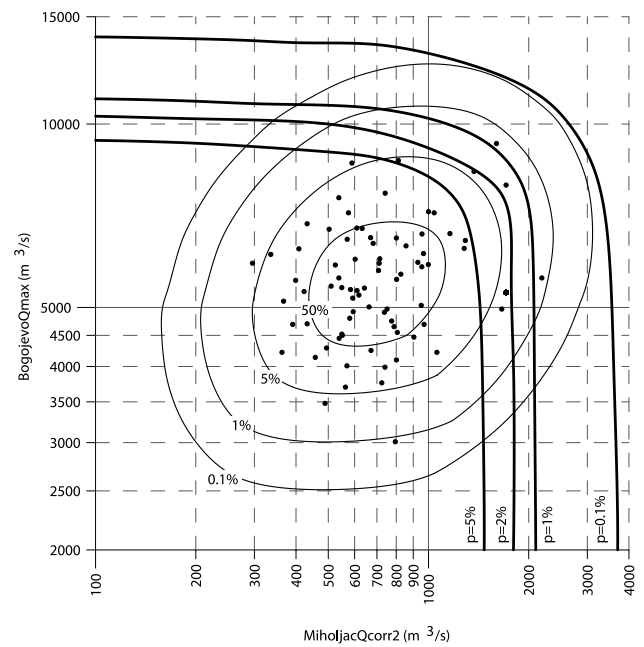


Fig. 10 Coincidence Q_{max} at Bogojevo GS on the Danube and Q_{corr2} at D. Miholjac GS on the Drava

Table 3 shows the quantitative indicators of the calculated flow values for different flood coincidence probabilities for the Danube and the tributaries, which are required for the definition of design level in the wider area of the analyzed confluences. The table shows the values for point 1 from the corresponding

exceedance line (P model) in the given combination of variables (see Sect. 2.4), and according to the survival copulas.

The proposed practical application of the results of probability of occurrence in a two-dimensional space using the P model and the C model (Table 3) are discussed below.

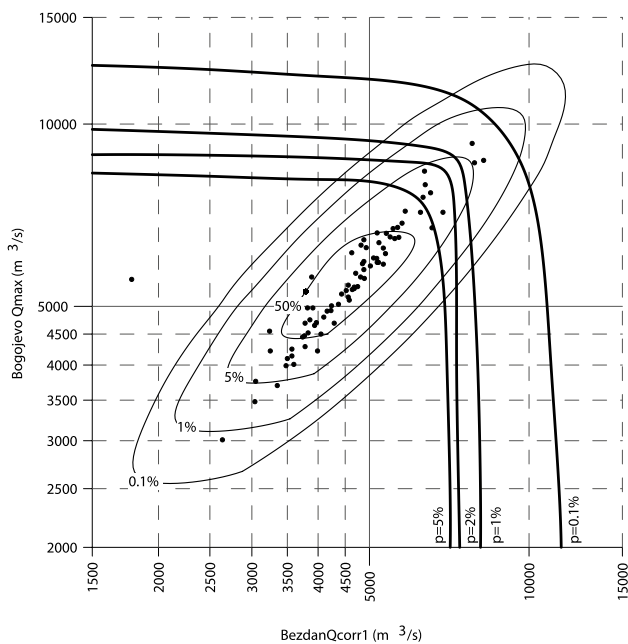


Fig. 9 Coincidence Q_{max} at Bogojevo GS and Q_{corr1} at Bezdán GS on the Danube

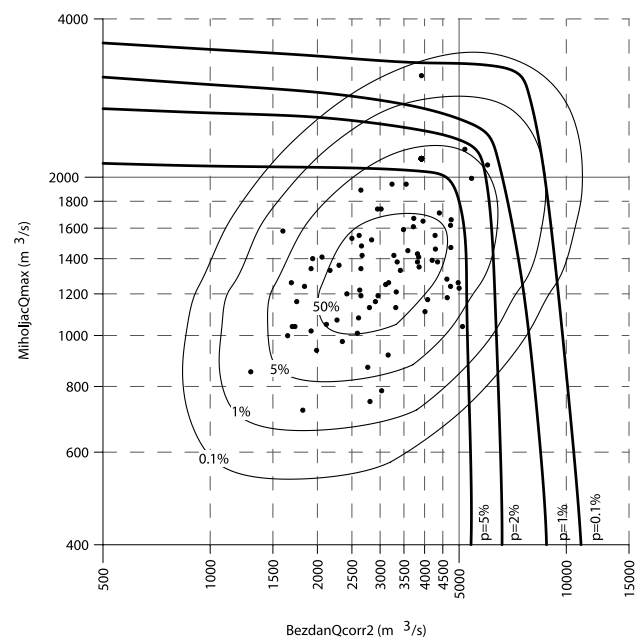


Fig. 11 Coincidence Q_{max} at D. Miholjac GS on the Drava and Q_{corr2} at Bezdán GS on the Danube

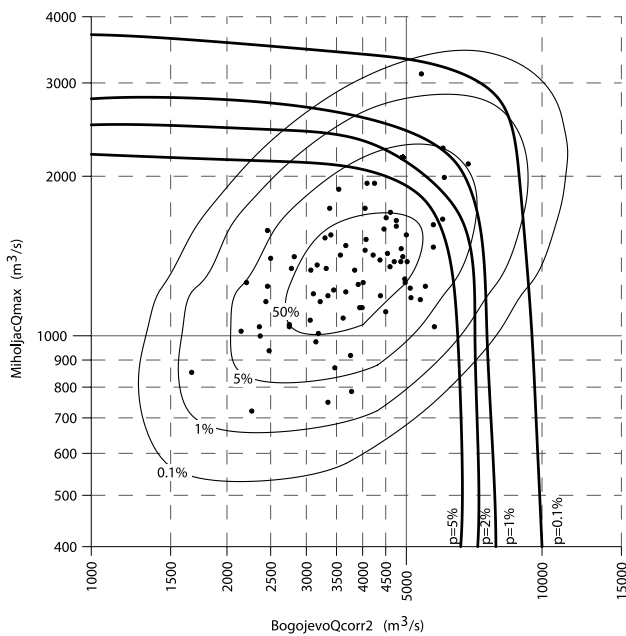


Fig. 12 Coincidence Q_{max} at D. Miholjac GS on the Drava and Q_{corr2} at Bogojevo GS on the Danube

The numerical values are outputs from the P model while the values in parentheses refer to the C:

1. For the reach of the Danube (NODE 1) upstream from the Dravaconfluence, in their interaction zone, the design level is the envelope of maximum levels obtained using:

- $Q_{max,1\%}^{BOG} = 9183(9219) \text{ m}^3/\text{s}$ and the corresponding flow upstream from the Drava confluence for

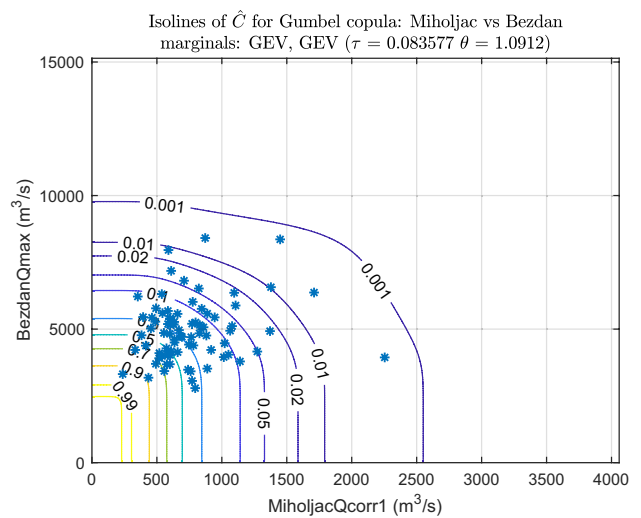


Fig. 14 Gumbel survival copula isolines for Q_{max} at Bezdán GS on the Danube and Q_{corr1} at D. Miholjac GS on the Drava

the 1% coincidence probability, from the graph $(Q_{corr1}^{BEZ}; Q_{max}^{BOG})$, $Q_{corr1}^{BEZ} = 6500(7390) \text{ m}^3/\text{s}$;

- corresponding flow downstream from the Drava confluence and the maximum annual flow upstream from the confluence for the 1% coincidence probability, from the graph $(Q_{corr1}^{BOG}; Q_{max}^{BEZ})$, $Q_{corr1,1\%}^{BOG} = 7300(7818) \text{ m}^3/\text{s}$ and $Q_{max,1\%}^{BEZ} = 8310(8248) \text{ m}^3/\text{s}$.

2. For the reach of the Drava upstream from the Danube confluence, the design level is obtained using:

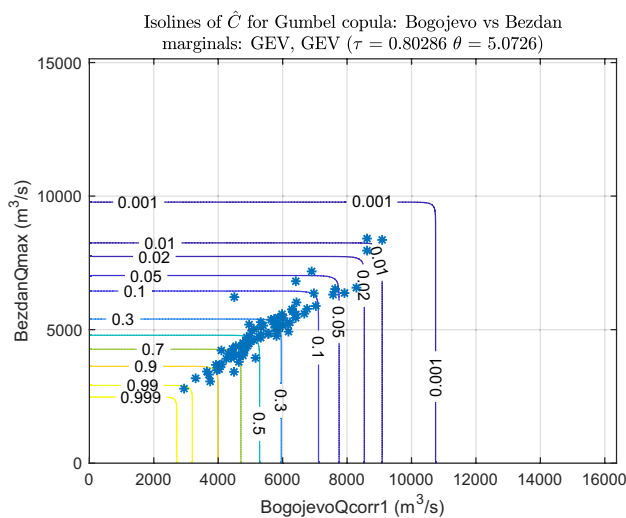


Fig. 13 Gumbel survival copula isolines for Q_{max} at Bezdán GS and Q_{corr1} at Bogojevo GS on the Danube

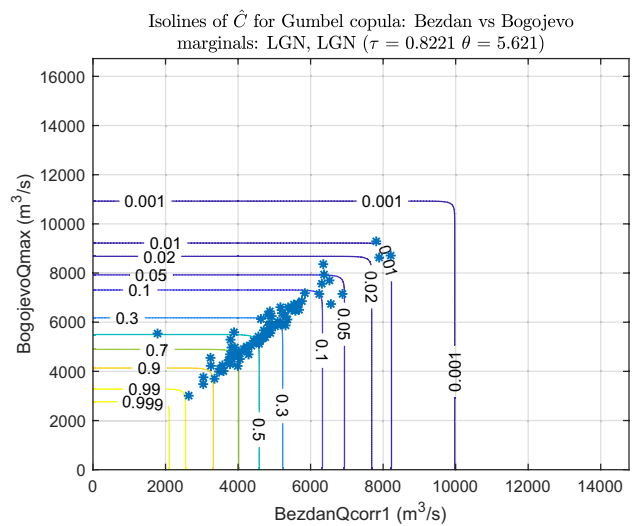


Fig. 15 Gumbel survival copula isolines for Q_{max} at Bogojevo GS and Q_{corr1} at Bezdán GS on the Danube

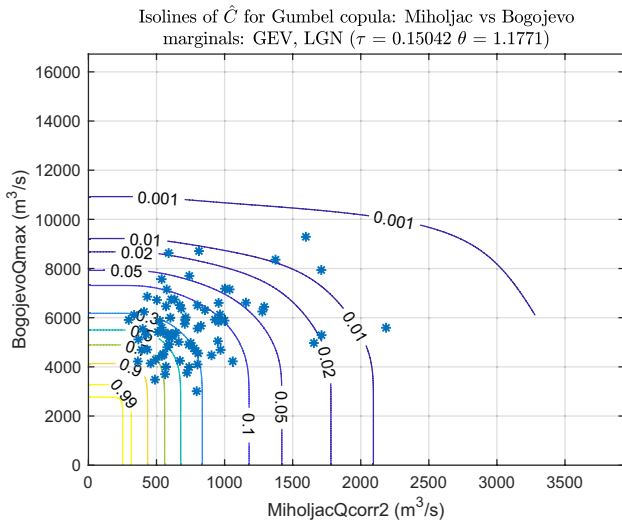


Fig. 16 Gumbel survival copula isolines for Q_{max} at Bogojevo GS on the Danube and Q_{corr2} at D. Miholjac GS on the Drava

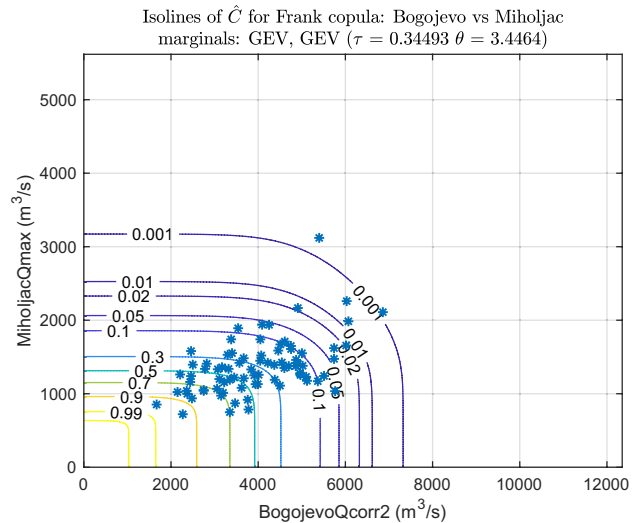


Fig. 18 Frank survival copula isolines for Q_{max} at D. Miholjac GS on the Drava and Q_{corr2} at Bogojevo GS on the Danube

- $Q_{max,1\%}^{BOG} = 9183$ (9219) m^3/s and the corresponding flow upstream from the confluence for the 1% coincidence probability, from the graph ($Q_{corr2}^{DM}; Q_{max}^{BOG}$), $Q_{corr2}^{DM} = 700$ (306) m^3/s ;
- corresponding flow of the Danube downstream from confluence and the maximum annual flow of the Drava upstream from the confluence for the 1% coincidence probability, from the graph ($Q_{max}^{DM}; Q_{corr2}^{BOG}$), $Q_{max,1\%}^{DM} = 2542$ (2526) m^3/s and $Q_{corr2}^{BOG} = 5300$ (1509) m^3/s .

3. For the reach of the Danube (NODE 2) upstream from the Tisa confluence, in their interaction zone, the design level is obtained using the following combinations of variables:

- $Q_{max,1\%}^{SLAN} = 10869$ (10530) m^3/s and the corresponding flow of the Danube upstream from the confluence for the same 1% coincidence probability, from the graph ($Q_{corr1}^{BOG}; Q_{max}^{SLAN}$), equal to $Q_{corr1}^{BOG} = 5000$ (6004) m^3/s ;
- corresponding flow of the Danube downstream from the Tisa confluence and the maximum annual flow of the Danube upstream from the confluence for the 1% coincidence probability, from the graph ($Q_{corr1}^{SLAN}; Q_{max}^{BOG}$), $Q_{corr1}^{SLAN} = 8600$ (8325) m^3/s and $Q_{max,1\%}^{BOG} = 9172$ (9232) m^3/s .

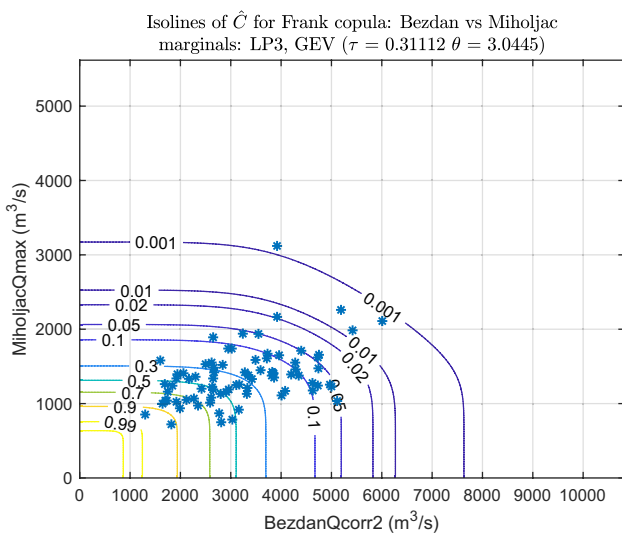


Fig. 17 Frank survival copula isolines for Q_{max} at D. Miholjac GS on the Drava and Q_{corr2} at Bezdan GS on the Danube

4. For the reach of the Tisa upstream from the Danube confluence, the design level envelope is obtained using:

- $Q_{max,1\%}^{SLAN} = 10869$ (10530) m^3/s and the corresponding flow of the Tisa upstream from the confluence for the 1% coincidence probability, from the graph ($Q_{corr2}^{SENT}; Q_{max}^{SLAN}$), $Q_{corr2}^{SENT} = 1200$ (629) m^3/s ;
- corresponding flow of the Danube downstream from the Tisa confluence and the maximum annual flow of the Tisa upstream from the confluence for the 1% coincidence probability, from the graph ($Q_{max}^{SENT}; Q_{corr2}^{SLAN}$), which in this specific case equals $Q_{max,1\%}^{SENT} = 3847$ (3997) m^3/s and $Q_{corr2}^{SLAN} = 8500$ (1245) m^3/s .

Table 3 Design flows for different flood coincidence probabilities $p=1\%$ of the Danube and its tributaries: the Drava, the Tisa, and the Sava

Mod.	NODE 1				Bogojevo GS				Donji Miholjac GS					
	Bezdan GS													
	$Q^{BEZ}_{max,p}$	$Q^{BOG}_{corr1,p}$	$Q^{DM}_{corr1,p}$	$Q^{BOG}_{max,p}$	$Q^{BEZ}_{corr1,p}$	$Q^{DM}_{corr2,p}$	$Q^{BOG}_{max,p}$	$Q^{DM}_{max,p}$	$Q^{BEZ}_{corr2,p}$	$Q^{BOG}_{corr2,p}$	$Q^{DM}_{max,p}$	$Q^{BOG}_{corr2,p}$	$Q^{DM}_{corr2,p}$	$Q^{BOG}_{corr2,p}$
P	8310	7300	500	9183	6500	700	9183	2542	4300	5300	2542	4300	5300	
C	8248	7818	226	9219	7390	306	9219	2526	1322	1509	2526	1322	1509	
	NODE 2				Slankamen GS				Senta GS					
	Bogojevo GS													
P	$Q^{BOG}_{max,p}$	$Q^{SLAN}_{corr1,p}$	$Q^{SENT}_{corr1,p}$	$Q^{SLAN}_{max,p}$	$Q^{BOG}_{corr1,p}$	$Q^{SENT}_{corr2,p}$	$Q^{SLAN}_{max,p}$	$Q^{SENT}_{max,p}$	$Q^{BOG}_{corr2,p}$	$Q^{SLAN}_{corr2,p}$	$Q^{SENT}_{max,p}$	$Q^{BOG}_{corr2,p}$	$Q^{SLAN}_{corr2,p}$	
C	9172	8600	1000	10869	5000	1200	10869	3847	4650	8500	3847	4650	8500	
	9232	8325	246	10530	6004	629	10530	3997	3674	1245	3997	3674	1245	
	NODE 3				Smederevo GS				Sremska Mitrovica GS					
	Slankamen GS													
P	$Q^{SLAN}_{max,p}$	$Q^{SMED}_{corr1,p}$	$Q^{SM}_{corr1,p}$	$Q^{SMED}_{max,p}$	$Q^{SLAN}_{corr1,p}$	$Q^{SM}_{corr2,p}$	$Q^{SMED}_{max,p}$	$Q^{SM}_{max,p}$	$Q^{SLAN}_{corr2,p}$	$Q^{SMED}_{corr2,p}$	$Q^{SM}_{max,p}$	$Q^{SLAN}_{corr2,p}$	$Q^{SMED}_{corr2,p}$	
C	11043	9700	1310	15128	9000	3200	15128	6700/6589	3000	7700	6700/6589	3000	7700	
	10680	9314	35	16602	6114	1710	16602	6927/6907	1164	5020	6927/6907	1164	5020	

5. For the final reach of the Danube (NODE 3) upstream from the Sava confluence, the design level is obtained using the following combinations of variables:

- $Q^{SMED}_{max,1\%} = 15128(16602) \text{ m}^3/\text{s}$ and the corresponding flow of the Danube upstream from the confluence for the same 1% coincidence probability, from the graph ($Q^{SLAN}_{corr1}; Q^{SMED}_{max}$), $Q^{SLAN}_{corr1} = 9000(6114) \text{ m}^3/\text{s}$;
- corresponding flow of the Danube downstream from the confluence and the maximum annual flow upstream from the confluence for the 100-year coincidence probability, from the graph ($Q^{SMED}_{corr1}; Q^{SLAN}_{max}$), $Q^{SMED}_{corr1} = 9700(9314) \text{ m}^3/\text{s}$ and $Q^{SLAN}_{max,1\%} = 11043(10680) \text{ m}^3/\text{s}$.

6. For the reach of the Sava upstream from the Danube confluence, the design flood level is obtained using the following combinations of variables:

- $Q^{SMED}_{max,1\%} = 15128(16602) \text{ m}^3/\text{s}$ and the corresponding flow of the Sava upstream from the confluence up to Sremska Mitrovica GS for the same 100-year coincidence probability, from the graph ($Q^{SM}_{corr2}; Q^{SMED}_{max}$), $Q^{SM}_{corr2} = 3200(1710) \text{ m}^3/\text{s}$;
- corresponding flow of the Danube downstream from the confluence and the maximum annual flow of the Sava upstream from the confluence for the 100-year coincidence probability, from the graph ($Q^{SM}_{max}; Q^{SMED}_{corr2}$), $Q^{SM}_{max,1\%} = 6581(6907) \text{ m}^3/\text{s}$ and $Q^{SMED}_{corr2} = 7700(5020) \text{ m}^3/\text{s}$.

The design of the flood control system first involves the determination of river levels using a hydrodynamic model. In addition to the proposed combinations of theoretical values shown in Table 3, one also needs to consider “the most likely event,” whose coordinates, according to Eq. 18, are shown in Table 4 for exceedance probability of 1%. It clearly shows that the deviations are significantly smaller and that the largest deviations take place in node 1, with the flow combinations $Q_{max,1\%}^{BOG}; Q_{corr2}^{DM}$, and in node 3, with the combinations $Q_{max,1\%}^{SMED}; Q_{corr2}^{SM}$.

The results also indicate that the P model and the selected Cs are in good agreement (Table 3) for all three nodes in the observed river reach in case of maximum flow at the entry profile (IN) and the corresponding flow at the exit profile (OUT) of the mainstem, with the adopted exceedance probability of 1%, and vice versa—in case of maximum exit profile flow and the corresponding entry profile flow. When maximum flow at the tributary profile (TR) and the corresponding mainstem exit profile flow (OUT) are combined, with the exceedance probability of 1%, and vice versa, the PROIL model and the copula procedure are again

Table 4 Comparison of coordinates of the most likely events in the PROIL model and the copula with percentage differences “Δ” for $p = 1\%$

	Combinations	C		P		Δmax(%)	Δcorr(%)
Node 1	$Q_{max}^{BEZ} - Q_{corr1}^{BOG}$	8159	8932	7900	8600	3	4
	$Q_{max}^{BOG} - Q_{corr1}^{BEZ}$	9105	8127	8800	7400	3	10
	$Q_{max}^{BOG} - Q_{corr2}^{DM}$	7558	1640	7500	1250	1	31
	$Q_{max}^{DM} - Q_{corr2}^{BOG}$	2153	5466	2100	6500	3	-19
Node 2	$Q_{max}^{BOG} - Q_{corr1}^{SLAN}$	9012	10072	8000	9600	13	5
	$Q_{max}^{SLAN} - Q_{corr1}^{BOG}$	10194	9316	10000	7900	2	18
	$Q_{max}^{SLAN} - Q_{corr2}^{SENT}$	9854	3280	9600	3200	3	3
	$Q_{max}^{SENT} - Q_{corr2}^{SLAN}$	3358	8728	3500	9400	-4	-7
Node 3	$Q_{max}^{SLAN} - Q_{corr1}^{SMED}$	10308	15262	10100	12700	2	20
	$Q_{max}^{SMED} - Q_{corr1}^{SLAN}$	16155	10575	14100	10050	15	5
	$Q_{max}^{SMED} - Q_{corr2}^{SM}$	15414	5304	14200	3950	9	34
	$Q_{max}^{SM} - Q_{corr2}^{SMED}$	6465	11672	5600	10600	13	9

in good agreement for maximum flows (a consequence of theoretical values being in good agreement), whereas the corresponding flows differ by about 50% (a consequence of theoretical values not being in good agreement).

4 Conclusions

The presented results lead to the conclusion that the methodology for calculating flood coincidence on the mainstem and a tributary is highly suitable for defining the theoretical flow values of given probabilities of occurrence on the mainstem reach downstream from the confluence, provided that the series of mean daily and maximum annual flows at both entry profiles are known. This provides a proper foundation for hydraulic calculations, which are used to define the design river levels for the design of flood control systems.

Observations of the selected reach of the Danube through Serbia have led to a conclusion that both the P model and the C model provide a high agreement of the results in case of the Danube portions upstream of the confluences with its tributaries, so the ultimate choice of the method is at the designer’s discretion.

However, in case of tributary portions, the results obtained using the C model were on average 40% lower than the P model results (and even 70% lower in case of the Tisa), which is why it is generally a better option to use the P model for the tributary portions to determine the combinations of flow calculations in order to calculate the design levels for the flood control system.

There are multiple benefits to be gained from the use of the multidimensional approach (P and C models):

1. Economic effects of the regulation of river reaches with tributaries

So far, for the selected flood risk, the practice has been to calculate the design flood downstream from the

confluence by simply adding the individual theoretical values of probabilities of occurrence equal to the adopted risk. The use of the P model and the C model has clearly shown that such a combination of values provides much larger risk coverage, which is not good in terms of cost-effectiveness of the construction.

2. Possibility to estimate maximum design flows at “insufficiently studied profiles”

As previously highlighted, in practice, design flood estimation for the adopted risk involves simple addition or subtraction of theoretical values of maximum annual flows, which correspond to the given risk if one value (upstream or downstream from the confluence, or on the tributary) in the node is unknown. Based on the established dependences in the nodes that constitute two entry profiles on the mainstem and a tributary and the exit profile on the mainstem, it is easy to calculate the unknown flow value on one profile if the other two are known.

3. Possibility to estimate the statistical significance of every historical flood in complex river systems

The established two-dimensional and multidimensional dependences allow the statistical significance of every historical flood to be estimated and thus pave the way for the planning of new flood control systems or the reconstruction of the existing ones.

4. Provision of real-time forecasting assistance to the operative forecasting service

The P model and the copulas can also provide assistance in the forecasting of maximum mainstem flow after the confluence with each tributary. If the forecasts of input hydrographs (by the Hydro Meteorological Services) are known, the established dependences (coincidences) can be used to calculate the theoretical value at the downstream profile and thus determine the hydrological scenario that can help implement an adequate flood control strategy.

Appendix 1. Marginal Probabilities $Q_{1\%}$

See Table 5, 6, and 7.

Table 5 Marginal probabilities $Q_{1\%}$ for node 1

Parameter	River	GS	PROIL $Q_{1\%}$ (m ³ /s)	Distribution	Copula $Q_{1\%}$ (m ³ /s)	Distribution
max	Danube	BEZ	8310	LP3	8248	GEV
corr1	Danube	BOG	9184	LP3	9089	GEV
corr1	Drava	DM	1810	LP3	1791	GEV
max	Danube	BOG	9183	LP3	9219	LGN
corr1	Danube	BEZ	7574	LP3	8230	LGN
corr2	Drava	DM	2061	LP3	2093	GEV
max	Drava	DM	2542	LP3	2526	GEV
corr2	Danube	BEZ	6384	LP3	6270	LP3
corr2	Danube	BOG	6919	LP3	6614	GEV

Table 6 Marginal probabilities $Q_{1\%}$ for node 2

Parameter	River	GS	PROIL $Q_{1\%}$ (m ³ /s)	Distribution	Copula $Q_{1\%}$ (m ³ /s)	Distribution
max	Danube	BOG	9172	LP3	9232	LGN
corr1	Danube	SLAN	9784	LP3	10,267	LP3
corr1	Tisa	SENT	3342	LP3	3432	LP3
max	Danube	SLAN	10,869	LP3	10,530	LGN
corr1	Danube	BOG	9105	LP3	9592	EV1
corr2	Tisa	SENT	3669	LP3	3587	GEV
max	Tisa	SENT	3847	LP3	3997	LGN
corr2	Danube	BOG	8198	LP3	8283	EV1
corr2	Danube	SLAN	10,357	LP3	10,412	LGN

Table 7 Marginal probabilities $Q_{1\%}$ for node 3

Parameter	River	GS	PROIL $Q_{1\%}$ (m ³ /s)	Distribution	Copula $Q_{1\%}$ (m ³ /s)	Distribution
max	Danube	SLAN	11,043	LP3	10,680	LGN
corr1	Danube	SMED	14,893	LP3	16,042	EV1
corr1	Sava	SM	5562	LP3	5128	GEV
max	Danube	SMED	15,128	LP3	16,602	EV1
corr1	Danube	SLAN	10,599	LP3	11,094	LGN
corr2	Sava	SM	5313	P3	5769	LP3
max	Sava	SM	6700	LP3	6927	EV1
corr2	Danube	SLAN	7849	LP3	7938	GEV
corr2	Sava	SM	6589	LP3	6907	EV1

Appendix 2. Copula Properties

See Table 8.

Table 8 Correlation coefficients, parameter of copulas, and goodness of fit p value

Node	GS	Combination	τ	ρ_s	C	θ	p
1	BEZ–DM	max–corr1	0.0836	0.1290	Gm	1.091	0.935
		corr2–max	0.3111	0.4464	Fr	0.304	0.716
		max–max	0.1033	0.1596	Cl	0.231	0.630
	DM–BOG	max–corr2	0.3449	0.5020	Fr	3.446	0.481
		corr2–max	0.1504	0.2254	Gm	1.177	0.806
		max–max	0.1787	0.2754	Cl	1.652	0.724
	BEZ–BOG	max–corr1	0.8029	0.9240	Gm	5.073	0.592
		corr1–max	0.8221	0.9417	Gm	5.621	0.177
		max–max	0.8272	0.9494	Gm	5.788	0.776
2	BOG–SENT	max–corr1	0.1689	0.2444	Gm	1.203	0.198
		corr2–max	0.2724	0.3939	Fr	2.612	0.480
		max–max	0.2139	0.3166	Gm	1.272	0.701
	SENT–SLAN	max–corr2	0.4817	0.6591	Fr	5.414	0.236
		corr2–max	0.3184	0.4665	Gm	1.467	0.370
		max–max	0.3935	0.5529	Gm	1.649	0.905
	BOG–SLAN	max–corr1	0.6871	0.8620	Gm	3.020	0.869
		corr1–max	0.6298	0.8024	Gm	2.701	0.906
		max–max	0.6927	0.8767	Gm	3.254	0.833
3	SLAN–SM	max–corr1	0.1111	0.1685	Gm	1.125	0.861
		corr2–max	0.1126	0.1739	Gm	1.127	0.630
		max–max	0.2928	0.4115	Gm	1.414	0.934
	SM–SMED	max–corr2	0.3314	0.4692	Gm	1.496	0.538
		corr2–max	0.3495	0.4909	Gm	1.537	0.707
		max–max	0.4666	0.6318	Gm	1.875	0.932
	SLAN–SMED	max–corr1	0.5386	0.7176	Gm	2.167	0.246
		corr1–max	0.6110	0.7994	Gm	2.571	0.893
		max–max	0.5559	0.7512	Gm	2.252	0.526

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Availability of Data and Material Data and materials may be available on the request.

Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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