SERBIATRIB '25

19th International Conference on Tribology

14 – 16 May 2025, Kragujevac, Serbia

PROCEEDINGS

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Serbian Tribology Society

University of Kragujevac
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Secretary: **Dragan Džunić**
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PHYSICS-BASED SIMPLE ANALYTICAL MODEL OF WATER FLOW THROUGH MICRO-POROUS FILTER

Nikola KOTORCEVIC[1,*], Fatima ZIVIC[1], Strahinja MILENKOVIC[2],
Nenad GRUJOVIC[1], Nikola MILIVOJEVIC[3]
[1]University of Kragujevac, Faculty of Engineering, Kragujevac, Serbia
[2]University of Kragujevac, Institute for information technologies, Kragujevac, Serbia
[3]Jaroslav Cerni Water Institute, Belgrade, Serbia
*Corresponding author: nikola.kotorcevic@uni.kg.ac.rs

Abstract: *In this paper, we studied governing wear mechanisms related to copper and copper-based alloys and composites considering their application for water filters. We reviewed the most important wear mechanisms of copper and its alloys including adhesion, abrasion, fatigue, erosion, cavitation with examples of copper application in tribological systems. Erosion corrosion has significant role in tribological behaviour of copper-based materials and their contacts in a fluid medium. We provided mathematical explanation of processes that occur during fluid flow through porous structures, including Darcy's law and Brinkman's equation. We developed the initial finite element (FE) model of water flow through porous medium to determine zones with increased possibility of wear occurrence. Computational model indicated that higher loads are present on the inflow side of the porous structure, thus the higher values of wear could be expected in that zone.*

Keywords: *copper, water filter, wear mechanism, analytical model, FE model*

1. INTRODUCTION

Copper and its alloys have wide range of application. Some of good characteristics of these materials are great thermal conductivity, wear resistance, low manufacturing cost and suitable and stable coefficient of friction [1], [2]. Because of this, they found application in many industrial manufacturing branches, such as: sleeve bearings and wheel gears [1]. Application of copper as a friction material is studied for purpose of fabrication of brakes and clutches. It is in a form of composite material with metal matrix. It is implemented in brakes and clutches of spacecraft, wind turbine, high-speed train, engineering machinery [2]. Among many others, application of copper can be also found in pipes for water transportation [3], including water filters for drinking water that have found use in many domestic devices.

Copper filters have diverse applications in different fields such as waste water treatment, drinking water purification, self-cleaning and bioadhesion control [4] [5]. Due to its good wetting properties and porosity, pure copper filters proved to be efficient water purificators with pronounced antibacterial properties that have been recognised from ancient times [4].

Creation of porous structures can be achieved using material extrusion printing technology or Fused Deposition Modelling (FDM) that belongs to additive manufacturing technologies, where filament is extruded through heated nozzle onto a plate and can form any type of customised internal geometry of the material structure [6].

The most common material used for FDM 3D printing technology is polylactic acid (PLA). Creation of composite materials used for filament creation can be achieved by addition of other materials into PLA. One such example is addition of copper particles. This Cu/PLA composite found application in many different fields, such as: filtration systems [7], [8], electronics [9], and biomedical field [10]. Composites with copper improve overall flexural strength, flexural modulus [11] and yield strength [12] of material. Also, composites with 90% of copper and 10% of PLA represent great antibacterial materials [13].

Different computational models have been developed and used to assist in characterizing and developing optimal porous materials structures, including analytical and mathematical models, numerical models such as finite element (FE) models and recent artificial intelligence (AI) supported numerical models [14], [15], [16].

This paper presents insights into the governing wear mechanisms related to copper and copper-based alloys and composites, including those used for filters. Analytical solutions for the fluid flow through porous media is discussed in relation to load distributions and their use in determining the zones with increased possibility for a significant wear occurrence. Simple finite element (FE) model of the water flow through the microporous filter is realised to provide initial insights into the important aspects of filtering systems.

2. WEAR MECHANISMS

Wear represents progressive loss of substance as a result of mechanical interaction between two surfaces in contact. The most important wear mechanisms are: adhesive, abrasive and fatigue wear.

Adhesive wear occurs as a result of two solid surfaces creating bond between each other. This transfer of material could lead to adhesive wear, if under the influence of relative motion forces particle of material to break apart. Adhesion presents starting point for many wear processes. As a product of wearing, debris is formed. It can oxidize or it can be formed as product of oxidization. This could further lead to creation of hard, abrasive particles which can remain in contact zone.

Abrasion is the wear mechanism where solid particles usually with higher values of hardness than base material, damage surface of material via various ways of cutting into it.

Fatigue wear occurs as the consequence of surface changes occurring over longer time that are induced by the surface strain due to the long-lasting external loading in a form of repetitive cycles where the loads acting on the frictional contacts are usually of lower values [3]. There is a critical limit of cycles before the surface damage occurs after which the contact becomes unstable.

Another wear mechanism is described as delamination. In this process surface layer is damaged due to crack nucleation and propagation in the subsurface level that is usually almost parallel to the surface. It is often observed in the case of composites and coatings [17]. Although many of those processes occur via solid interface, they are also present in fluids, especially in the case of high impact velocities and high energy flows.

Erosion and cavitation have been shown to have a significant influence on the materials in contact with different fluids depending on the fluid velocities and pressure distribution. Material erosion in fluids arises with impact of fluid droplets upon material surface, while cavitation represents surface damage as a result of vapor or gas bubbles collapse. Both mechanisms usually occur in fluids with high velocity [3]. Studies indicate that the most important factors that impact erosion wear are flow velocity and impact angle. It is elaborated that with higher velocity of particle, higher kinetic energy is generated upon impact on material [18].

If the contact environment is fluid, chemical wear can be significant depending on the type of fluids and contact materials. Chemical wear occurs as result of chemical reaction between materials in contact. In many cases chemical processes are not isolated from mechanical ones and usually complement each other [3].

2.1 Copper wear mechanisms

Many researchers investigated tribological behaviour of copper and its alloys in solid form. It was observed that friction coefficient of most metallic surfaces was decreasing with rising of surface temperature. Frictional heat has an important influence on the tribological behaviour and failure of sliding components. At lower sliding speeds dominant wear mechanisms were abrasive and adhesive wear, while at higher speeds it was oxidation wear and delamination wear [19].

Others report that at temperatures of 25°C dominant wear mechanism were mild oxidation, adhesion and abrasive wear, at 400°C to 600°C severe oxidation, abrasive and adhesive wear and at 800°C oxidation and delamination wear [2].

As for fluid medium, the most dominant wear mechanism for copper pipes used for freshwater transportation is erosion corrosion. It represents damage of the material surface usually concentrated in local zones, due to corrosive fluid flow. This means that protective layer of the copper is mechanically removed from the surface and that bare copper metal is subjected to the corrosive effect of water. To achieve good performance, copper requires formation of protective films on its surface, mainly Cu_2O. Without the presence of such protective layers, copper degradation due to corrosion is much faster (**Figure 1**) [3].

Processes that affect erosion corrosion of copper can be mechanical as influenced by flow rates, cavitation, gas bubble, particulate impingement or electrochemical in nature where the influencing factors are chemistry, temperature and corrosion reactions of water. These two processes are combining and create synergic relationship. Processes that are detected as potential factor for erosion corrosion propagation are gas bubble cavitation within transporting pipes and solid particles which are acting abrasively upon material surface. It is also observed that erosion corrosion occurs mainly in hot water systems. In cold water, it is usually because of the higher water pressure.

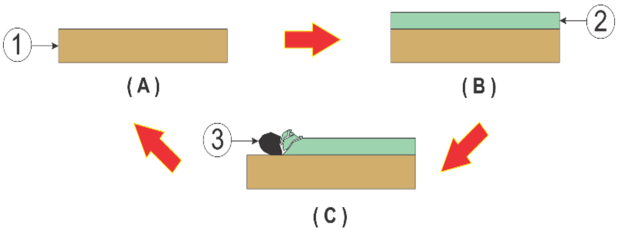

Figure 1. Cyclic process of film removal: A- Corrosion acting on pure substrate, B- Formation of passivating layer, C- Loss of film, 1- base material, 2- passivating film, 3- abrasive particle

Cavitation is defined as spontaneous nucleation, growth and collapse of gas or vapor bubbles when local hydraulic pressure is below gas saturation pressure or vapor pressure of the water. Solid particles can end up in water transportation system as residues from water treatments or subsequent later contamination. With their presence, processes like abrasion, erosion and wear can occur. Magnitude of abrasive wear is depending on the hardness of solid particles and material surface [3].

3. ANALYTICAL SOLUTIONS FOR THE FLUID FLOW THROUGH POROUS MEDIA

Darcy's law is often used equation for explanation of fluid flow through porous media. It was derived from Navier-Stokes equations, experimentally explained by Henry Darcy and it represents formulation of conservation of momentum. First results came from experiments of water flow through beds of sands and, naturally, its application is very common in hydrogeology but is also used to explain flow of oil, water and gas through petroleum reservoirs. Darcy's law represent relationship between discharge rate through a porous medium, the viscosity of the fluid, and pressure drop over a given distance, as described in equation 1 and **Figure 2** [20]:

$$Q = \frac{-k\cdot A\cdot(p_2-p_1)}{\mu\cdot L} \qquad (1)$$

Where Q [m^3/s] is total discharge of volume per time, k [m^2] - intrinsic permeability of the medium, A [m^2] – cross-section area of the flow, $(p_2 - p_1)$ [Pa] – total pressure drop, μ[Pas] – viscosity, L[m] – length.

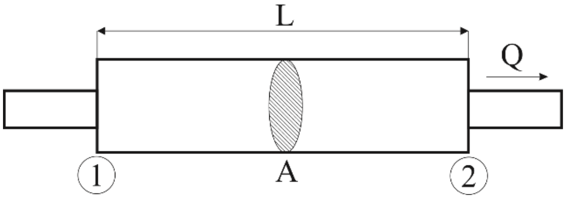

Figure 2. Directions of Darcy's law flow: L – length over which pressure is dropping, A – cross-section of the flow, Q – total discharge

By dividing both sides with area, general form of Darcy's law is acquired, represented via equation 2:

$$q = \frac{-k}{\mu}\cdot\nabla p \qquad (2)$$

Where q is flux discharge per unit area [m/s], ∇p is pressure gradient vector [Pa/m] and "–" sign indicates that fluid is flowing from higher to lower pressure.

Application of Darcy's law is possible only for the viscous, slow flow that has lower values of Reynolds number which is non-dimensional parameter expressed with equation 3:

$$R_e = \frac{\rho\cdot v\cdot d}{\mu} \qquad (3)$$

Where ρ represents density of water [kg/m^3], υ[m/s] is specific discharge, d[m] is representative grain diameter of porous media. Darcy's law may be considered applicable for flow regimes with Reynolds number up to 10 [20]. Those conditions indicate that flow can be considered laminar. Turbulent flow can be assumed at higher values of Reynolds number (≈2000) or if fluid transportation system contains obstructions downstream as shown in **Figure 3**. In porous media, such as porous water filters, every strut within the porous structure can be observed as the obstruction to water flow, meaning that any inhomogeneity (e.g. possible changes in the strut thickness over the whole strut length due to fabrication method) is a potential zone where turbulent micro-flows can appear within the small, restricted zones. In the case of many such turbulent micro-flows, it can significantly influence the overall fluid behaviour, flow and acting on the filter material.

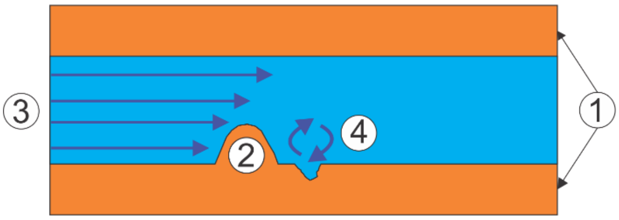

Figure 3. Schematic representation of fluid flow: 1 - pipe, 2 – obstruction, 3 – laminar flow, 4 - turbulence

Darcy's law is used to describe average flow on macroscopic level and create relation between pressure gradients and flow velocity vector in the fluid phase, taking gravity into account, via equation 4 [21]:

$$V = -\frac{k}{\mu}\cdot(\nabla p - \rho g) \qquad (4)$$

Where V is fluid velocity vector, k is permeability tensor, ∇p is pressure gradient, ρ is fluid mass density and g is gravity vector.

In his study, Brinkman was looking for more appropriate modification of this equation in order to describe viscous force. Navier-Stokes equations of viscous, incompressible fluid yielding can be considered in this case [22].

This led to the formulation of Darcy-Brinkman equation or Darcy's law with Brinkman correction, described in equation 5 [23]:

$$V = -\frac{k}{\mu} \cdot (\nabla p - \rho g) + k \cdot \nabla^2 V \ (5)$$

Where the last part of equation ($k\nabla^2 V$) was introduced in order to take into account the viscous shear effect that opposes the free flow through the porous structure [23].

4. SIMPLE FINITE ELEMENT (FE) MODEL OF THE WATER FLOW THORUGH THE POROUS FILTER

We used COMSOL Multiphysics software to simulate simplified water flow through porous media. 2D model geometry included microporous structure with width of 200 mm and height of 250 mm. Inflow was from the left side of the filter, while outflow was on the right, as explained in **Figure 4**.

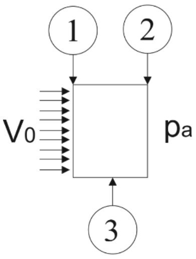

Figure 4. Boundary conditions of the model: 1 – inflow, 2 – outflow, 3 – walls, V_o – fluid velocity, p_a – atmospheric pressure

FEM calculations used mesh with free 2D quadrant as represented in **Figure 5**.

Speed of fluid inflow was V_0 = 0,3 m/s, porosity of medium was ε_p = 0,8, permeability k = 10^{-12} m^2, density ρ = 1000 kg/m^3, dynamic viscosity was μ = 10^{-3} Pa·s, temperature T = 293,15 K. Parameters were adopted from the recommended values of water. Flow was considered Darcian and equation used for solving was Brinkman equation. Our particular interest was load and velocity that occurred under these conditions. Pressure gradient is shown in **Figure 6** and velocity gradient in **Figure 7**.

It should be noted that this simple model considers stationary system with constant fluid velocity. In the case of other type of geometry, such as variable pipe diameters over its length, the model would provide different velocity models with relevant distributions.

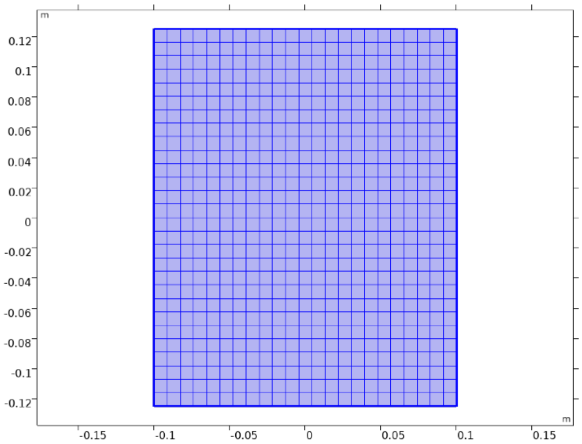

Figure 5. Mesh used for FE model analysis

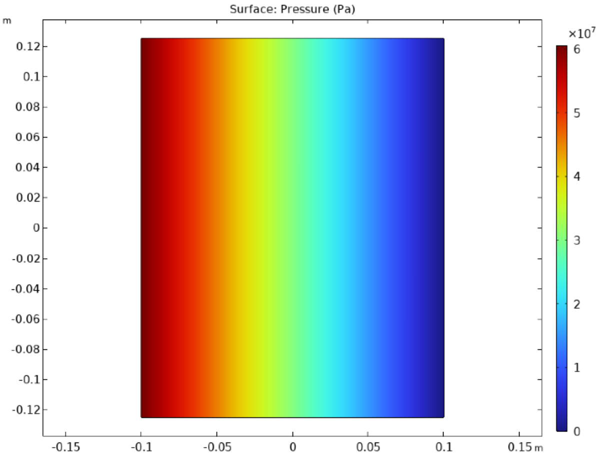

Figure 6. Pressure gradient

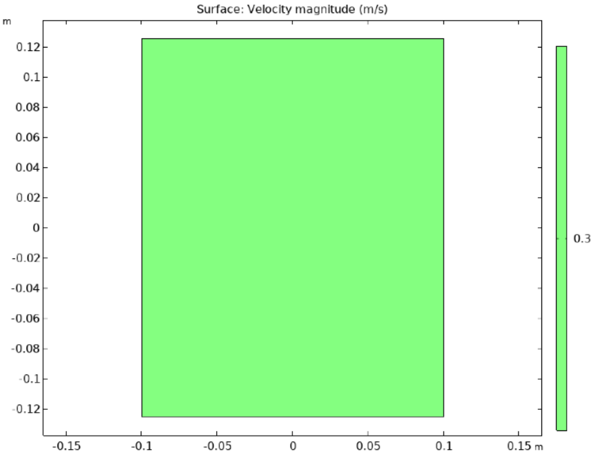

Figure 7. Velocity gradient

Further research will be focused on the shape and size of the pores within the micro-porous filter and their influence on the load and pressure distribution and modes. Computational models can efficiently shorten the discovery process of new materials, including topological features of the porous structures targeting the desired mechanical properties, stability during its function and enhanced durability.

5. CONCLUSION

Copper has the application in real systems that are used for fluid transportation, as well as for water filters. Since its protective layer is subjected to degradation, special interest should be put to the circumstances under which it occurs. Since modelling of real-life flow through porous structure was complicated from mathematical perspective, simplified models are beneficial for the initial analysis. This should be used as starting point in creation of the more sophisticated models that are closer to reality. Our research indicated that the areas with higher values of load should have more attention in the design of the porous structure from wear aspects, since in those zones it is more likely that higher wear rates would occur. Direct consequence of the increased wear rate is formation of wear debris that can block filter pores over time, depending on the wear mechanisms that are governed by the material and the shape and thickness of the internal porosity, whereas the thickness is one of the important influences.

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ORCID iDs

Nikola KOTORCEVIC 0000-0002-5664-9832
Fatima ZIVIC 0000-0003-2509-187X
Strahinja MILENKOVIC 0000-0002-5664-9832
Nenad GRUJOVIC 0000-0002-8765-2196
Nikola MILIVOJEVIC 0000-0001-5330-1219

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