ICSSM 2021
Proceedings

8th International Congress
of the
Serbian Society of Mechanics

June 28-30, 2021
Kragujevac, Serbia

Hellenic Society
for Theoretical &
Applied Mechanics
(HSTAM)

СДРМ
СРПСКО ДРУШТВО ЗА РАЧУНСКУ МЕХАНИКУ
SERBIAN SOCIETY FOR COMPUTATIONAL MECHANICS

MINISTRY OF EDUCATION,
SCIENCE AND TECHNOLOGICAL DEVELOPMENT

The 8th International Congress of the Serbian Society of Mechanics Kragujevac, Serbia, June 28-30, 2021

Editors
Professor Miloš Kojić, academician
Professor Nenad Filipović

Technical Editor
Đorđe Dimitrijević

Technical Assistant
Miloš Anić

Proofreader
Neda Vidanović Miletić

Publisher
Serbian Society of Mechanics,
9/1 Kneza Miloša, 11000 Belgrade, Serbia

Press
"Grafo Ink", Kragujevac

Impression
100 copies

Year
2021

Organizers

- **Serbian Society of Mechanics (SSM)**
- **Faculty of Engineering, University of Kragujevac**
- **Faculty of Mechanical Engineering, University of Belgrade**
- **Faculty of Technical Science, University of Novi Sad**
- **Faculty of Mechanical Engineering, University of Niš**
- **Hellenic Society of Theoretical and Applied Mechanics**
- **Institute of Information Technology Kragujevac**
- **University of Kragujevac**

with the support of

- **Ministry of Education, Science and Technological Development**
- **Serbian Academy of Sciences and Arts**

and

- **Serbian Society of Computational Mechanics**

Sponsor

- **TSI**

Table of Contents

Welcome Message

Dear colleagues,

It is a great pleasure for us to welcome you all at *the* ***8th International Congress of the Serbian Society of Mechanics*** in Kragujevac, Serbia Well-known for its culture, history and industrial heritage, Kragujevac was the first capital of modern Serbia and the place where the first constitution in the Balkans was proclaimed. Today, we are more than proud to say that Kragujevac is also becoming one of the scientific capitals in the region.

In this very difficult time of the COVID-19 pandemic, we decided to make this congress a hybrid event combining physical and online sessions, so that everyone interested can join us despite the obstacles we have all been facing for more than a year now.

8th International Congress of the Serbian Society of Mechanics aims to bring together leading academic scientists, researchers and research scholars to exchange and share experiences and research results on various aspects of *Theoretical and Applied Mechanics*. It will bring an interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, theories, algorithms, as well as practical challenges encountered and solutions adopted in the fields of *Classical Mechanics, Solid and Fluid Mechanics, Computational Mechanics, Biomechanics, Applied Mathematics and Physics, Structural Mechanics and Engineering.*

The Congress is organized by the Serbian Society of Mechanics (SSM) in partnership with: Faculty of Engineering, University of Kragujevac, Faculty of Mechanical Engineering, University of Belgrade, Faculty of Technical Science, University of Novi Sad, Faculty of Mechanical Engineering, University of Niš, Hellenic Society of Theoretical and Applied Mechanics, Institute of Information Technology Kragujevac, University of Kragujevac, with the support of the Serbian Ministry of Education, Science and Technological Development, Serbian Academy of Sciences and Arts and Serbian Society for Computational Mechanics.

Six distinguished plenary speakers will deliver lectures:

1. Prof. Georgios E. Stavroulakis – Technical University of Crete, Greece
2. Prof. Themis Exarchos – Ionian University, Corfu, Greece
3. Prof. Mihailo R. Jovanović – University of Southern California, USA
4. Prof. Ricardo Ruiz Baier – Monash University, School of Mathematics, Clayton, Australia
5. Dr Božidar Jovanović – MISANU, Serbia
6. Dr Marko Janev – MISANU, Serbia

The Congress encompasses six main topics: General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies, Biomechanics, Control and Robotics, Interdisciplinary and Multidisciplinary Problems.

Also, there are four Mini-Symposia:

- M1: 5th Serbian-Greek Symposium on Advanced Mechanics
  Chairs: Prof. Georgios Stavroulakis, President of HSTAM, Greece; Prof. Nenad Filipović, President of SSM, Serbia
- M2: Turbulence
  Chair: Prof. Đorđe Čantrak, University of Belgrade, Serbia
- M3: Mathematical Biology and Biomechanics
  Chair: Dr. Anđelka Hedrih, MI SANU, Serbia
- M4: Nonlinear Dynamics
  Prof. Julijana Simonović, University of Niš, Serbia

Within the Congress, we are also very proud to organize the 5th Serbian-Greek Symposium on *Current and Future Trends in Mechanics*. The Symposium is organized by the Serbian Society of Mechanics (SSM) and the Hellenic Society of Theoretical and Applied Mechanics (HSTAM).

This year, *8th International Congress of the Serbian Society of Mechanics* received more than 150 high-quality research papers. Each paper was reviewed and ranked by at least 2 professors and scientists in the program and the scientific review committee. As a result of the strict review process and evaluation, the committee selected 120 research papers.

We must also say that the conference would certainly not have been so successful without the efforts of many people who were actively engaged in organization of such a major nationally and internationally recognized academic event. We give our special gratitude to the members of the program and scientific review committee as well as to all chairs, organizers and committee members for their dedication and support.

On behalf of the Organizing Committee, we wish you all a pleasant stay in Kragujevac and a productive conference.

Chairs:

Prof. Nenad Filipović, ***president of SSM, University of Kragujevac***
Prof. Miloš Kojić, ***Serbian Academy of Sciences and Arts***

Technical Program

Monday 28 June 2021

08:45 - 09:15	Opening Ceremony - Welcome speech: **Prof. Nenad Filipović**, President of SSM, Conference Co-Chair **Prof. Miloš Kojić**, full member of SASA, Conference Co-Chair **Nikola Dašić**, Major of Kragujevac City **Prof. Ivica Radović**, State Secretary, Ministry of Education, Science and Technological Development, Serbia **Prof. Dobrica Milovanović,** Dean of Faculty of Engineering, Kragujevac

09:15 - 09:45	Keynote speaker: Topic: **Analysis of a New Mixed Formulation for Hyperelasticity Using Kirchhoff Stress** **Prof. Ricardo Ruiz Baier**, Monash University, School of Mathematics, Clayton, Australia Chair: Hedrih A.

Session M.1A: 09:45-11:00

Biomechanics (part I)

Chairs: Kojić M., Geroski V.

M.1A.1 – Extension of our computational model for the left ventricle tissue to include hypertrophy *– Kojić M.*

M.1A.2 – Coupled Ohara-Rudy numerical model for heart electro-mechanics – *Geroski V., Milošević M., Milićević B., Simić V., Filipović N., Kojić M.*

M.1A.3 – Electromyography detection of muscle response in musculus quadriceps femoris of elite volleyball players on different exercises – *Radaković R., Peulić A., Kovač S., Simojlović M., Filipović N.*

Session M.1B: 09:45-11:00

Mechanics of Solid Bodies (part I)

Chairs: Mastilović S., Dunić V.

M.1B.1 – Remarks on discreteness of the nanoscale fragmentation mass distribution – *Mastilović S.*

M.1B.2 – Size-effect modeling of Weibull Jc cumulative distribution function based on a scaling approach – *Mastilović S., Đorđević B., Sedmak A.*

M.1B.3 – Material parameters identification of concrete damage plasticity material model – *Rakić D., Bodić A., Milivojević N., Dunić V., Živković M.*

M.1B.4 – Using of gap element for contraction joints modeling in seismic analysis of concrete arch dams – *Živković M., Jović N., Pešić M., Rakić D., Milivojević N.*

M.1B.5 – Finite element analysis of effects of multiple defects on welded joint integrity – *Aranđelović M., Sedmak S., Jovičić R., Sedmak A., Radaković Z.*

11:00 - 11:30	Coffee Break

Session M.2A: 11:30-13:00

Interdisciplinary and Multidisciplinary Problems (part I)

Chairs: Sedmak A., Nikolić D.

M.2A.1 – Noise induced dynamics of earthquake nucleation model – *Kostić S., Vasović N.*

M.2A.2 – Nonlinear landslide dynamics – *Kostić S., Vasović N.*

M.2A.3 – Computational mechanics – welding joint as a case study – *Jeremić L., Sedmak A., Sedmak S., Martić I.*

M.2A.4 – Experimental electrochemotherapy using novel design single needle device – *Cvetković A., Cvetković D., Milasinović D., Jovičić N., Miailović N., Nikolić D., Mitrović S., Filipović N.*

M.2A.5 – Cavitation diagrams for merchant ships using four blade b series propellers – *Veg M., Kalajdžić M.*

M.2A.6 – Microfluidic lab-on-chip system development for cell culture cultivation – *Milivojević N., Živanović M., Nikolić D., Jovanović Ž., Šeklić D, Nikolić M., Filipović N.*

Session M.2B: 11:30-13:00

Mechanics of Solid Bodies (part II)

Chairs: Rakić D., Obradović A.

M.2B.1 – New pipe ring tensile specimen for pipeline material fracture assessment – *Trajković I., Rakin M., Milošević M., Sedmak A., Međo B.*

M.2B.2 – Mass minimization of an AFG Timoshenko cantilever beam with a large body placed eccentrically at the beam end – *Obradović A., Mitrović Z., Zorić N.*

M.2B.3 – On concentrated surface loads and the flat punch contact problem in strain gradient elasticity – *Zisis T., Gourgiotis P., Georgiadis H.*

M.2B.4 – Geometric optimization of shaft transition zone based on stress-strain analysis of nature inspired design – *Atanasovska I., Momčilović D.*

M.2B.5 – A comparative analysis of fatigue behaviour between S355J2+N and Strenx 700 steel grade – *Živković M., Milovanović B., Dišić A., Jovičić G., Topalović M.*

M.2B.6 – Linear transient analysis of spatial curved Bernoulli – Euler beam using isogeometric approach – *Jočković M., Nefovska Danilović M.*

13:00 - 14:00	Buffet Lunch

Session M.3A: 14:00-15:00

Interdisciplinary and Multidisciplinary Problems (part II)

Chairs: Milošević M., Šušteršič T.

M.3A.1 – Analysis of atherosclerotic plaque in carotid arteries by using convolutional neural networks – *Arsić B., Đorović S., Anić M., Gakovć B., Končar I., Filipović N.*

M.3A.2 – Structural condition assessment and rehabilitation of 'Karpos' system bridge – *Milošević M., Živković S., Marković Branković J., Marković M.*

M.3A.3 – Epidemiological predictive modelling of COVID-19 spread – *Šušteršič T., Blagojević A., Cvetković D., Cvetković A., Lorencin I., Baressi Šegota S., Car Z., Filipović N.*

M.3A.4 – In vitro and in silico testing of stent device – *Nikolić D., Saveljić I., Filipović N.*

8th International Congress of Serbian Society of Mechanics
Kragujevac, Serbia, June 28-30, 2021

NONLINEAR LANDSLIDE DYNAMICS

Srđan Kostić[1], **Nebojša Vasović**[2]

[1] Jaroslav Černi Water Institute
Jaroslava Černog 80, 11226 Belgrade, Serbia
e-mail: srdjan.kostic@jcerni.rs
[2] Faculty of Mining and Geology,
University of Belgrade, Đušina 7, 11000 Belgrade
e-mail: nebojsa.vasovic@rgf.bg.ac.rs

Abstract

In present paper we analyze two different mechanical models of landslide dynamics. The first model actually represents a deterministic model of infinite slope evolution, whose rich dynamics is observed for different friction properties of the sliding surface, illustrated by the state parameter β, and under the effect of the introduced time delay t in time-dependent shear stress. Results of the analysis performed indicate that solely the increase of time delay could further invoke the occurrence of complex dynamical behavior: quasiperiodic oscillations and deterministic chaos. Besides the evident effect of the assumed delayed shear stress, there is also a record of significant effect of friction parameter β and the higher level of amplitude of initial stress perturbation due to the possible external seismic effect. In particular, the increase of parameter β, i.e. the reduction of friction force in steady state (minor direct velocity effect) and weak evolutionary velocity effect drives the system to less complex dynamical regimes regardless of the level of the assumed time delay. The second model represents a model of a block sliding down the slope, under the stochastic effect of groundwater level oscillations. Developed model indicates that displacements along the slope are observed simultaneously with the change of groundwater level.

Key words: landslide, nonlinear dynamics, time delay, noise, friction

1. Introduction

Landslides represent features of contemporary geodynamical processes which commonly affect the infrastructural facilities: roads of different categories (hihways, regional roads, etc.), railways and pipelines, as well as different hydrotechnical structures: dams, structures for shore protection and embankments. Research on the possible impact of natural landslides on the design, construction andexploitation of the aforementioned structures, including the analysis of the cnditions for occurence of slope instability in different earth structures, is on e of the most important research task in the phase of deisgn and supervision during the construction. Considering this, deep understanding of the mechanicsm of landslide process, which includes the conditions for the occurrence of slope instability and the regime of sliding in time, is of uttermost importance for engineering practice.

There are several different approach on the analysis of landslide dynamics. One approach assumes the use of the well established limit equilibrium methods, which have a long traidtion of application in engineering practice, and which enable quick and reliable estabimation of the slope stability. Another approach includes the application of finite element method, which enable numerical quantifiction of slope stability, and which provides more accuracte results than LEM method. Although both of these methods are being use today in enginnering practice as common methoda for estimation fo slope stability (and hence the analysis of the conditions of landslides to occur) and, as such, are incorporated in commerical and open-source engineering softwares, they do not enable clear unbderstanding of the mechanism od the sliding process itself: what are the triggering conditions and what regime of dynamics could be expected? Answers to these questions is possible to obtain by invoking the theory of nonlinear dynamics, which enable the clear indetifiction of the triggering conditions of landslide occurrence and the indeiniftication of different dynamical regimes, under the conidition that nonlinear landslide model is set in the proper way. The first attempt in this direction was made by Davis [1], who proposed a model for accumulation slide. In this paper, Davis analyzed dynamics of two-block system on an inclined plane, described by three ordinary differential equations, as an analog to the landslide motion. As a result, he determined an approximate stability criterion for surging motions of accumulation slides, depending on the slope geometry, strength of slope materials and the ratio of masses of accumulation and feeder slides. On the other hand, Morales et al. [2] proposed a model for blocks sliding down a slope as a vriant of the Burridge-Knopoff model, where the shear stress described by the local potential is replaced by a constant tangential force induced by gravity. In this paper, authors propose three different friction laws: Coulomb-like, cube and piecewise-linear friction. They constructed front waves for a piecewise linear friction force and provide explicit formulas for the wavespeed and the wave form. Authors of this paper also made some research on this topis. For instance, Kostić et al. [3] analyzed dynamics of a single-block model on a inclined slope with Dieterich-Ruina friction law under the variation of time delay and initial shear stress. They determined conditions for occurrence of different dynamical regimes, including the conditions for the onset of deterministic chaos. Also, Kostić et al. [4] proposed modified version of the model by Morales et al. [2] by including the effect of the groundwater level oscillations. Their results indicate the occurrence of irregular dynamics similar to the observed one, and the correlation between the movement down the slope and increase of groundwater level.

In present paper, authors propose two nonlinear models of landlside dynamics, with the following goals: (1) to determine the conditions for landslide to occur; (2) to identify different regimes of landslide dynamics. In particular, authors invoke methods of nonlinear dynamics, which are then used to examine effect of perturbation of different parameters controlling the stability and establish the conditions for transitions between different dynamical regimes.

2. Landslide model with time delay and stress perturbation

The first landslide model is formulated starting from the model of infinite slope evolution, with the incorporated time delay in shear stress [5]:

$$\begin{aligned}
{}^{d\tau_d}\!/_{dT} &= -\lambda e^{V}[s(T-t)-\tau_{d0}-(1-\beta)\nu]+\frac{e^{-\nu}}{\kappa}(\gamma-\tau_d)\\
{}^{d\nu}\!/_{dT} &= \frac{e^{-\nu}}{\kappa}(\gamma-\tau_d)\\
{}^{d\delta}\!/_{dT} &= e^{\nu}
\end{aligned} \tag{1}$$

where $\tau_d=\tau/A$ (stress), $\nu=ln(V/V_0)$ (velocity), $\delta=u/h$ (displacement), $T=V_0t/h$ (time), $\tau_{d0}=\tau_0/A$, $\kappa=\rho V_0^2/A$, $\gamma=\rho ghsin\alpha/A$, $\beta=B/A$ and $\lambda=h/L$, g is gravitational constant (9.81m/s^2), α is the slope angle, ρ is the mass density, h is the thickness of the overlying soil, τ is the shear strength along the sliding surface, V and u are the velocity and displacement of the block, respectively, A represents material constant, dependent on rock type, pressure, temperature and sliding velocity [6], τ_0 represents the threshold shear stress at some reference sliding velocity V_0, B is an empirical constant, depending on the properties of soil, L is a characteristic slip distance comparable to a typical asperity length. The delayed response of the friction is modeled through the introduced time delay t, which could be interpreted as delayed effect of precipitation, as a common triggering factor of slope instability, which corresponds well with the results of laboratory test on fluid-saturated clay with low sliding rates, performed by Skempton [7].

The analysis is conducted numerically using Runge–Kutta 4th order numerical integration method. At each instance, the parameters held constant are awarded values near the equilibrium point. The dynamics of the block for $\beta<1$ and for t in range [0.1-10] is shown in Figure 1a. Numerical approach indicates complex behavior, with small areas of quasiperiodic and chaotic motion (Fig. 1a). For $\beta=1$, block velocity decreases for a first few values of time delay, suppressing the motion and leading eventually to equilibrium state of the block. For $\beta>1$, introduced time delay invokes the chaotic behavior of the model under study (Figure 1a), which is confirmed by irregular aperiodic oscillations (Figure 1b).

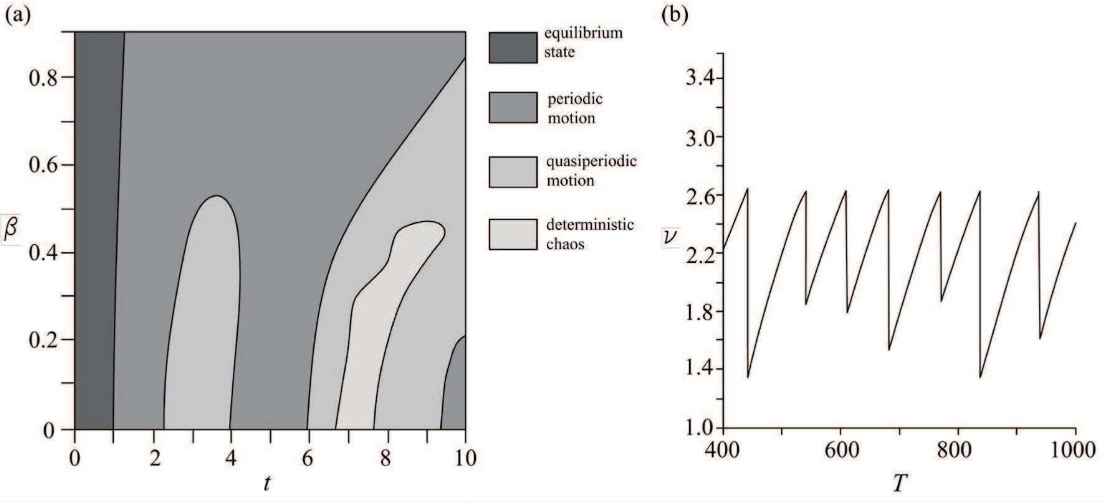

Figure 1. (a) Bifucation diagrams t-β illustrating transitions between equilibrium state, periodic and quasiperiodic motion and deterministic chaos; (b) Time series $v(T)$ for $\beta=1.1$ and $T_d=0.1$. Other parameter values are held constant: $s_0=1.0$, $\lambda=1.5$, $k=2.0$, and $\gamma=1.0$.

Different dynamical regimes occur if one assumes periodic perturbations of the initial shear stress due to external earthquake triggering effect, even without the included time delay:

$$\mu(T) = \tau_{d_0} + \delta_s \sin(\omega_s T) \tag{2}$$

where δ_s and ω_s represent the constant oscillation amplitude and the angular frequency, respectively. In particular, for $\beta < 1$, by perturbing only the shear stress μ, while the other parameters are held constant for creeping equilibrium state, landslide exhibits oscillatory behavior for any frequency value in the range [0.1–2.0]. On the other hand, for $\beta>1$, landslide velocity increases for frequencies higher than $\omega_s=4.9$.

Besides the aforementioned analysis, the periodic perturbation of parameter τ_{d0} could be assumed in system (1) with included time delay t, for limit amplitude value and $\omega_s = 0.5$. In this case, attractors of the system (1) are shown in Fig. 2a for $\beta < 1$ and for t in range [0.1–10].

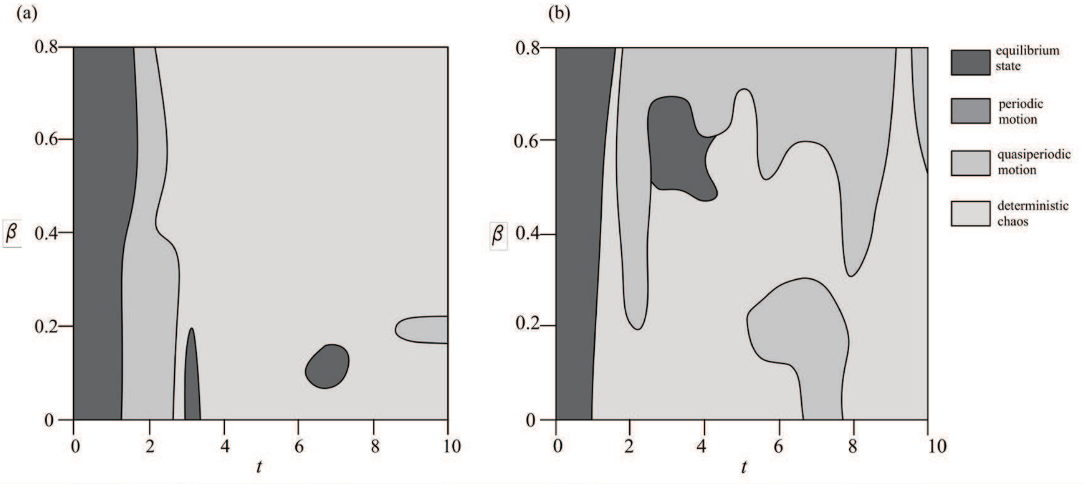

Fig. 2. Bifurcation diagrams t-β illustrating transitions among periodic motion, quasiperiodic motion and deterministic chaos, under the variation of μ, with δs = 1.0 (a), .δ_s = 0.5 (b). Diagrams are constructed for step size equal 0.1 for both t and β. For both diagrams other parameter values are: τ_{d0} = 1.0, λ = 1.5, κ = 2.0, γ = 1.0, ω_s = 0.5.

Diagram in Fig.2b is constructed for oscillation amplitude δ_s = 0.5. Physically possible solutions are obtained for β > 1. In particular, for τ = 0.1, system (1) exhibits chaotic dynamics, if the perturbation amplitude is δs ≤ 0.18. Deterministic chaos is further corroborated by broadband noise in Fourier power spectrum and by positive value of maximal Lyapunov exponent (Figure 3).

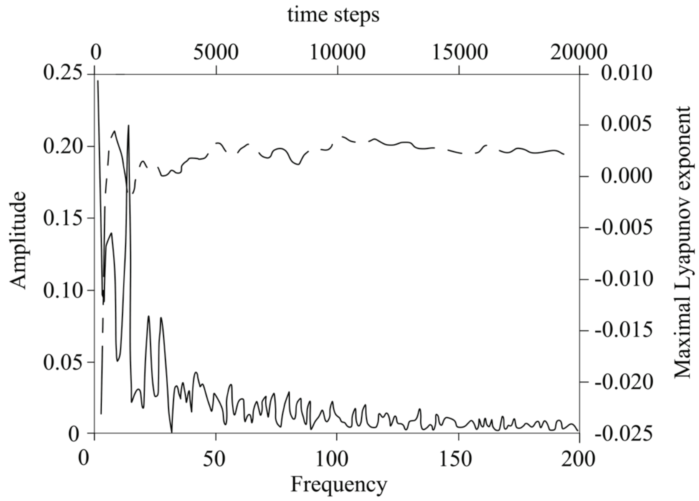

Figure 3. Continuous broadband noise in Fourier power spectrum and maximum Lyapunov exponent (converging to positive value λ_{max}=0.002). Other parameter values are: β=1.1, s_0=1.0, λ=1.5, κ=2.0 and γ=1.0.

3. Stochastic landslide model

As for the second landslide model, we start from a model defined by Morales et al. [39], with the included effect of groundwater level (GWL) oscillations:

$$\frac{du}{dt} = f * v$$
$$\frac{dv}{dt} = -\frac{k}{1+v} + G + h * g - u$$
$$\frac{dg}{dt} = a\sin\theta + b\cos\theta + z(t) - c \cdot g(t-\tau) - g^3 + z(t) \tag{3}$$
$$dz = -\frac{z}{\varepsilon}dt + \sqrt{\frac{2D}{\varepsilon}}dW$$
$$\frac{d\theta}{dt} = \omega$$

where *u* stands for displacement, *v* is the velocity of the potentially unstable block, *G* is the gravity constant, *F(v)* is the friction term, $F(v)=\kappa/(1+v)$. Parameters *k*, *f* and *h* represent parameters for tuning the effect of friction, displacement scales and influence of groundwater level oscillations, respectively.

One should note that groundwater level oscillations are included in Eq. (3) in the following way. Kostić et al. (2019) proposed the following dynamical system for the GWL oscillations:

$$\dot{g} = a \cdot sin(\omega t) + b \cdot cos(\omega t) - c \cdot g(t-\tau) - g^3 + z(t)$$
$$dz(t) = -\frac{z}{\varepsilon}dt + \sqrt{\frac{2D}{\varepsilon}}dW \tag{4}$$

where *a* and *b* are Fourier coefficients, while parameter *c* controls the effect of autocorrelation properties on the GWL dynamics. Variable *g* stands for the groundwater level. Variable *z(t)* represents an Ornstein-Uhlenbeck process, and term$(2D/\varepsilon)^{1/2}$ dW represents stochastic increment of independent Wiener process, i.e. dW satisfy: E(dW) = 0, where E() denotes the expectation over many realizations of the stochastic process. The noise correlation time ε and the intensity of noise D are parameters that can be varied independently. Colored noise generated by Ornstein-Uhlenbeck process with this parametrization is referred to as power-limited colored noise. Qualitatively, this colored noise could be ascribed to the potential effect of the groundwater inflow from the background.

It is shown in the work of Kostić et al. [2019] that GWL oscillations recorded at four different piezometers in Serbia are in fact random by nature.

Standard bifurcation analysis is performed numerically, using Runge-Kutta 4th order method Results obtained are shown in Fig. 4. It is clear from Fig.4 that by increasing either time delay or delay strength one can see the transition from fixed point to periodic motion when no oscillations are assumed in the starting system (ω=0). On the other hand, if oscillations are assumed in the starting deterministic system (ω=1), there are transitions among different types of oscillation: from low frequent and moderate amplitude oscillations, over high frequent and high amplitude oscillations to quasiperiodic behavior. Moreover, one could spot the transitions between different dynamical regimes is gradual, i.e. when equilibrium point becomes unstable, then small-diameter limit cycle occurs, followed by the increase of diameter with the increase of the bifurcation parameter value or time delay, implying the possible occurrence of a supercrictical direct Andronov-Hopf bifurcation.

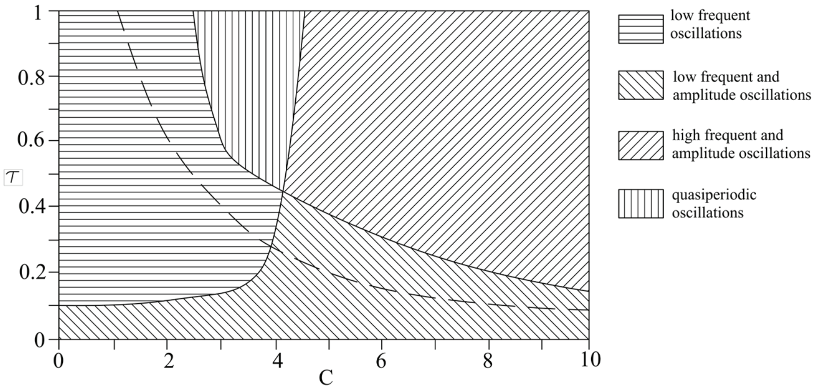

Fig. 4. Bifurcation diagrams *τ-c*: (a) ω = 0, (b) ω = 1. When ω = 0, there is a clear transition from equilibrium state to oscillatory regime (periodic oscillations), while for ω = 1 rich dynamical behaviour occurs. A

Results of numerical analysis of system (3) indicate that GWL oscillations of groundwater level are followed by corresponding changes in landslide displacement (Fig. 5). One should note that oscillations for GWL and slope displacements are obtained using the positive part of the solutions u and g of the system (11):

$$u^{+}, g^{+}(t) = \max(u(t), g(t), 0) = \begin{cases} u(t), g(t), & \text{if } u(t), g(t) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

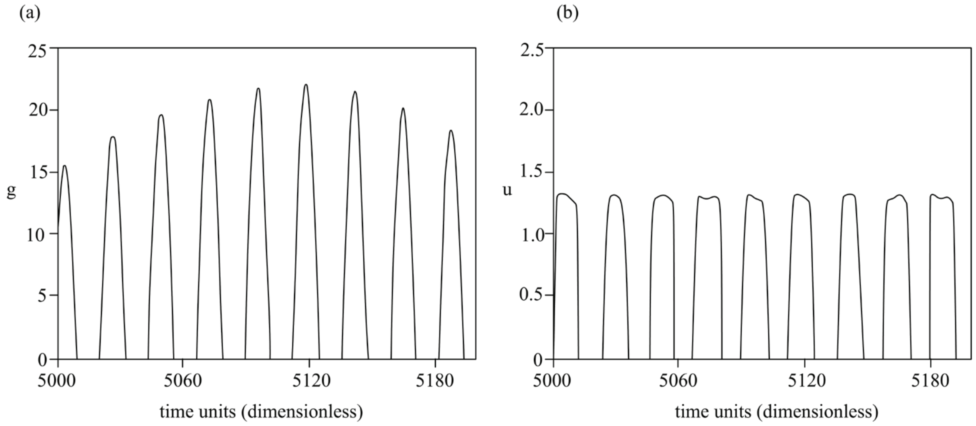

Fig. 5. Groundwater level oscillations and corresponding changes in block displacements. Results are obtained for the following constant parameter values: k = 0.01, f = 0.07, h = −1, a = b = ω = 0.5, c = −1, ε = 1, τ = 10, D = 0.1, G = 1. Initial conditions are set away for the equilibrium state: u = 0.001, g = 0.710, V = 0.401, z = 0.001, θ = 0.002. Results are given after neglecting the transients (first 5000 time units).

4. Conclusions

In present paper we analyze dynamics of two landslide models, which could help one gain new insights into the mechanism governing the motion of the unstable slope. In the first model, which represents an extension of the famous Burridge-Knopoff model, we study the effect of three significant contributing factors: (1) impact of the assumed time delay in the change of shear stress; (2) effect of friction along the sliding surface, through parameter β, (3) influence of the assumed perturbation of initial shear stress. The first contributing factor actually represents the effect of the state variable, illustrating different state of the sliding surface: existence of asperities, pore pressure effect etc. The effect of the assumed time delay t reveals itself in the occurrence of complex dynamical behavior with the increase of t, changing from equilibrium state through periodic and quasiperiodic motion up until the appearance of deterministic chaos. On the other hand, one could find as very interesting the effect of parameter β. This parameter actually represents the ration of two parameters relating to the soil properties, β=B/A. As it could be seen in Fig. 6 parameter A is the measure of direct velocity effect, i.e. it measures the rate of friction increase in the steady state of the slope. On the other hand, parameter B measures the rate of friction decrease during the motion along a slope, the so-called evolutionary velocity effect.

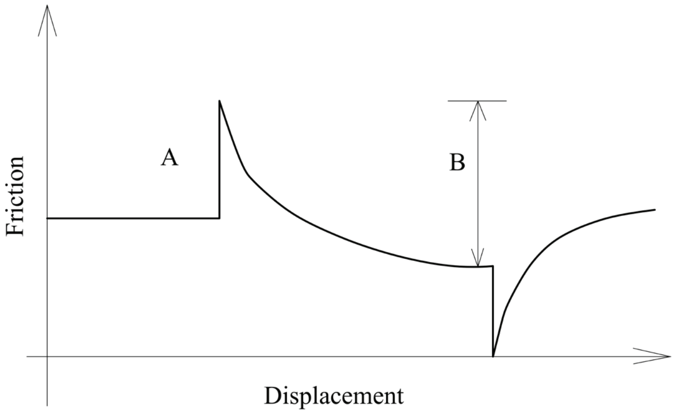

Fig. 6. Assumed friction law along the sliding surface and the friction parameters A and B.

The analysis performed indicate that the increase of β leads to the occurrence of less complex dynamical behavior, leading eventually the system to equilibrium state, regardless of the assumed level of time delay in shear stress. This means that in case when the effect of friction along a sliding surface is minor (e.g. high pore pressure effect, groundwater level along a sliding surface, etc.), shear stress with included time delay alone cannot induce the onset of complex behavior, only the transition from equilibrium state to periodic motion. Regarding the effect of the external seismic input, our analysis confirmed that no bifurcation could be induced by the sole impact of these external perturbations. However, the co-effect of external perturbations and delayed shear stress could lead to occurrence of deterministic chaos, but only for the assumed high-amplitude perturbations.

In the second model, we analyze the effect of stochastic groundwater level oscillation on the dynamics of a landslide model. It is shown that motion along an unstable slope occurs simultaneously with the rise of groundwater level.

Further research on this topic should expand the dynamical analysis of a landslide model with stochastic effect of groundwater level oscillations. In particular, one should further investigate bifurcation types and dynamical regimes which occur in this model.

References

[1] Davis, R.O. *Modelling stability and surging in accumulation slides*. Engineering Geology 33, 1-9, 1992.

[2] Morales, J.E., James, G., Tonnelier, A. *Travelling waves in a spring-block chain sliding down a slope*. Physical Review E 96, 012227, 2017.

[3] Kostić, S., Vasović, N., Franović, I., Jevremović, D., Mitrinović, D., Todorović, K. *Dynamics of landslide model with time delay and periodic parameter perturbation*s. Communications in Nonlinear Science and Numerical Simulation 19, 3346-3361, 2014.

[4] Kostić, S., Stojković, M., Guranov, I., Vasović, N. *Revealing the background of groundwater level dynamics: Contributing factors, complex modelling and engineering applications*. Chaos, Solitons and Fractals 127: 408-421, 2019.

[5] Chau KT. *Landslides modeled as bifurcations of creeping slopes with nonlinear friction law*. International Journal of Solids and Structures;32:3451–64, 1995.

[6] C. Scholz, *The mechanics of earthquakes and faulting*, Cambridge University Press, United Kingdom, 471 p.

[7] Skempton AW. *Residual strength of clays in landslides, folded strata and the laboratory*. Geotechnique ;35:3–18, 1985.